Ion sound turbulence in a magnetoplasma

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An experimental investigation of current-driven ion sound turbulence in a collisionless magnetized plasma column is presented. Frequency and wave vector spectrum of the saturated instability are analyzed. The cross-power spectral density of probe signals shows that waves are localized in $k$ space both in the electron drift direction and along the instability cone of angle $\theta = \arccos(c_e/c_i) \leq 90^\circ$. In spite of good plasma uniformity, azimuthal asymmetries in wave propagation are noticed. The self-consistent electron distribution function deviates from a shifted Maxwellian.

I. INTRODUCTION

Although the current-driven ion acoustic instability has been investigated for many years, the understanding of the saturation mechanism and the turbulent plasma properties is still rather limited. Recently, attention has been focused on collisionless plasmas in a magnetic field such as occur in the magnetosphere or in toroidal$^{2-4}$ and linear$^{5-8}$ laboratory devices. In order to characterize the instability and its effects on the plasma, a rather detailed knowledge of the particle distribution and turbulent fields is required. Since few experiments supply complete information, a careful study of current-driven ion sound turbulence has been undertaken.

The instability is generated by drawing a field-aligned current through an essentially uniform, collisionless, magnetized plasma column of dimensions large compared with the wave correlation length. The turbulence spectrum is analyzed by performing three-dimensional cross-power spectrum measurements of probe signals. While previous studies$^{9,10}$ in collisional plasmas showed dominant wave propagation in the electron drift direction, we observe ion acoustic waves propagating mainly along the instability cone of angle $\theta = \arccos(c_e/c_i) \leq 90^\circ$, where $c_e$ and $c_i$ are electron drift and ion sound speed, respectively. The propagation direction varies in different frequency regimes. Thus, commonly employed correlation techniques in which all frequency components are analyzed at once give very complicated correlation surfaces.$^7$ Although, in principle, Fourier transforms of the correlation surfaces in space and time yield equivalent information,$^11$ the cross spectral analysis is preferred due to its ease of operation.

Spectral analysis by microwave$^6$ or laser$^9$ scattering techniques would also yield a complete picture of turbulence spectrum if all directions and magnitudes of wave vectors $k$ would be investigated, but this is usually not done.

Particle distribution functions are analyzed with probes and energy analyzers. It is found that the self-consistent electron distribution function in the collisionless but turbulent plasma is not a Maxwellian shifted by $|\nu_e| = |j|/ne$, where $j$ and $n$ are current density and electron density, respectively. Rather, the distribution function appears asymmetric with a wider velocity spread in the drift direction. This is interpreted by a wave drag exerted on slower electrons rather than the formation of energetic electrons.

A most important yet difficult measurement is that of dc electric fields parallel to the field-aligned current. Such measurements are essential for establishing the existence of anomalous resistivity.$^1$ The common practice to derive $E_e$ from gradients in the floating potential may be applicable to collisional plasmas$^{12}$ but can lead to large errors in non-Maxwellian plasmas.$^{13}$ Gradients in the plasma potential are more reliable yet difficult to establish in a turbulent plasma, even with emissive probes. We have therefore developed a new diagnostic technique for measuring $E_e$, to be described in more detail in a companion paper.$^{14}$ A test electron beam is injected along or opposite to the electron drift and the electric field is estimated from the test particle velocity change. The observed dc fields are found to be negligibly small compared with the fluctuating fields, implying that particle scattering dominates over runaway acceleration.

Section II describes the experimental setup, followed by the measurement results of plasma parameters in Sec. III and of the wave properties in Sec. IV. Section V gives a brief summary since further observations on the particle dynamics in the turbulent plasma are given in the companion paper.$^{14}$

II. EXPERIMENTAL ARRANGEMENT

The experiment is performed in a linear, magnetized discharge plasma$^{15}$ of characteristic parameters: density $n_e \approx 10^9$ cm$^{-3}$, temperature $T_e \approx 10$ T$_i \approx 2$ eV, magnetic field $B_0 \approx 130$ G, collision frequencies $\nu_{ce}/\omega_{pe} \approx 10^{-4}$, $\nu_{ce}/\omega_{pe} \approx 10^{-3}$, gases argon, helium, uniform plasma diameter 12 cm, length 80 cm. The plasma is produced by volume ionization with fast electrons (100 eV). These electrons are emitted from a filamentary tungsten cathode located in a large diameter (1 m) source chamber in which the plasma is nearly unmagnetized ($B_0 \approx 10$ G). High plasma quiescence and uniformity are obtained not only in the source chamber but also in the adjoining long (120 cm) experimental region of strong uniform magnetic field. The metallic end wall of the source chamber forms the discharge anode (see Fig. 1). The typical discharge current is $I_e \approx 2$ A.

The plasma in the experimental region is terminated by an end electrode (15 cm diam) from which current
Typically $\omega/2\pi = 100$ MHz, $\omega_{ce}/2\pi = 480$ MHz, $\theta_e = 15^\circ$, which yields $n_e = 4 \times 10^4$ cm$^{-3}$. With $\omega_{pe} > \omega_{ce}$ the electrons are strongly magnetized, whereas the ions are not. In the presence of current flow ($I_p = 250$ mA, $V_s = 5$ V) the density decreases to $n_e = 2 \times 10^3$ cm$^{-3}$. Since the production rate due to ionization by primary electrons (100 eV energy) is not significantly changed, the density decrease indicates an enhanced ion loss rate. The cone measurements are performed in the central flat part of the density profile, so that errors due to density gradients or reflections from boundaries are negligible. With increasing current the peak density fluctuations rise from a minimum value $\delta n/n \approx 1\%$ up to $\delta n/n \approx 30\%$, which causes a broadening of the field pattern although the cone structure remains readily visible.

The electron distribution function and drift velocity are obtained from one-sided plane Langmuir probes. Both single and differential probes are used which can be rotated with respect to the magnetic field $B_0$. The characteristics of the single probe (2.5 mm x 4 mm) are traced out in 500 $\mu$sec near the end of a 3 msec anode pulse which is sufficiently long to establish steady-state conditions. The probe current is sampled and averaged with a boxcar integrator and fed on-line into a PDP 11 computer which differentiated and plotted the distribution function.

Figure 2(a) shows typical $I-V$ characteristics at dif-

\[ \omega_{pe}^2 = \omega_{ce}^2 - \omega^2 \cdot \left( \omega_{pe}^2 \sin^2 \theta_e / \omega^2 - 1 \right) \cdot \]

![Diagram](image)

**FIG. 1.** Schematic arrangement of experimental setup. The discharge plasma is generated in the fringing magnetic field region, the experiment is performed in a uniform field ($B_0 = 150$ G, $n_e = 2 \times 10^6$ cm$^{-3}$, $T_e = 10$ $T_0 = 2$ eV, $Ar = 2 \times 10^5$ Torr).

can be drawn along the magnetic field. However, it was found that in this arrangement the electron drift was limited ($v_e < v_p$) and the plasma potential closely followed the applied potential. By placing a fine-mesh grid in the front part of the experimental region, the column was divided into a double plasma device consisting of a test region (80 cm length) and the source region. Electrons drawn to the end anode of the test section are easily supplied from the source region of $n_e$, in general, higher density. Large electron drifts ($v_e < 0.3 v_p$) at relatively small electrode potentials ($V_e < 10$ V) are obtained with this arrangement. The end anode voltage is, in general, pulsed, and all data are sampled and time averaged with a box-car integrator.

The plasma diagnostics are performed with a variety of probes: (i) Planar, one-sided, rotatable, Langmuir probes for determining the relative density, the electron drift, background and test electron distributions, (ii) rf probes for resonance cone measurements which yield the absolute density, (iii) pairs of identical small cylindrical Langmuir probes (0.125 mm diam, 1.5 mm length) for three-dimensional cross correlation measurements, and (iv) a retarding grid ion energy analyzer for determining background and test ion distributions.

In order to observe the effects of turbulence on the particles, test electrons and ions are injected into the plasma in the form of pencil beams along and across the magnetic field, respectively. The particle sources and detectors are described in detail in a subsequent paper.\textsuperscript{14}

**III. PLASMA PROPERTIES**

Here, we briefly describe how the basic plasma parameters necessary to establish the instability condition are obtained. The effects of the turbulence on the plasma properties is discussed in more detail in Ref. 14.

Since absolute density measurements with Langmuir probes in magnetized plasmas are subject to large errors probes are only used for relative density and fluctuation measurements. The absolute density is obtained from the angle of rf resonance cones excited with a small rf probe. From the measured cone angle $\theta_e$, applied frequency $\omega$ and cyclotron frequency $\omega_{ce}$, the electron plasma frequency $\omega_{pe}$ is derived according to

\[ \omega_{pe}^2 = \left( \omega_{ce}^2 - \omega^2 \right) / \left( \omega_{ce}^2 \sin^2 \theta_e / \omega^2 - 1 \right) \cdot \]

![Figure 2(a)](image)

**FIG. 2.** (a) Sampled $I-V$ characteristics of one-sided Langmuir probes facing either toward anode (left-hand traces) or cathode (right-hand). With increasing end anode voltages $V_e$, the saturation currents differ markedly indicating current flow in the plasma. (b) Normalized electron drift velocity calculated from Eq. (2) and the measured saturation currents of Fig. 2(a).
different anode voltages with the probe surface normal facing either against the electron drift $v_\phi$ ($\phi = 0^\circ$, toward source) or with $v_\phi$ ($\phi = 180^\circ$, toward anode). In the absence of current flow the electron saturation currents in either direction are not quite identical, which can be explained by the shadow effect of the probe on the primary electron flow from the source. As the anode current is increased by raising $V_a$, the probe saturation currents become consistently unbalanced. If the distribution function were a drifting Maxwellian, the current ratio would be given by the expression

$$\frac{I_e}{I_\phi} = \frac{\exp(-x^2) + \sqrt{\pi} x (1 + \text{erf}x)}{\exp(-x^2) - \sqrt{\pi} x (1 - \text{erf}x)},$$

(2)

where $I_e/I_\phi$ are saturation currents toward the anode, cathode, respectively, and $x = v_\phi/v_e$ is the ratio of electron drift to thermal speed. Under this assumption the measured current ratio yields a drift velocity as shown in Fig. 2(b). The drift velocity saturates at $v_\phi \approx 0.3 v_e$ with increasing anode voltage. The end anode current behaves similarly, i.e., it saturates with anode voltages $V_a \geq 5$ V.

A more careful inspection of the derivative of the probe traces shows that the distribution function is not quite a Maxwellian. Combining the data from the two opposite directions, one obtains the parallel distribution function $f_\phi(v_\phi)$ as shown in Fig. 3. It indicates an asymmetry, i.e., the peak of $f_\phi$ is shifted to lower velocities than that of the dashed symmetry line for the wings (corresponding to $v_\phi$ for Maxwellian). There is, of course, no reason to expect a Maxwellian distribution since in the present case the mean free path for electron-electron ($l_{ee} \approx 2 \times 10^4$ cm) and electron-neutral collisions ($l_{en} \approx 10^3$ cm) far exceeds the dimensions of the device ($L \lesssim 10^2$ cm). Scattering off turbulent waves is the dominant randomizing process which surprisingly, closely establishes an equilibrium distribution. In the absence of such collisions one may not expect to find a significant electron flux directed away from the anode. When drawing saturation current, the anode is near the plasma potential and reflects only few slow electrons. Volume ionization contributes only a negligibly small electron current on the order of the ion saturation current. Essentially all the electrons drawn from the anode are supplied from the source, but on their passage they are significantly scattered. This will be shown by test particle measurements in Ref. 14.

The width of the distribution function is observed to increase with $V_a$ indicating that the electron temperature increases ($\Delta T_e/T_e = 2.5$, $T_e \approx 1$ eV). Both the magnitude and the mechanism for the temperature increase have to be considered carefully. For example, one cannot take a one-sided probe trace and evaluate $T_e$ by a simple semilogarithmic plot since the drift causes vastly different "apparent" temperatures in either direction of $v_\phi$. More reliable is a measure of the $1/e$ width of the shifted distribution function as shown in Fig. 3 or, as will be described, a measurement of the speed $c_s = (T_e/m_e)^{1/2}$ using test ion acoustic waves. The mechanism for the temperature increase is not due to electrons acquiring energy from a parallel dc electric field and anomalous resistivity (turbulent heating) but due to randomization of the initial streaming velocity of the injected electrons. This conclusion is drawn largely from the results of test particle studies and careful measurements of parallel fields. In general, electrons do not gain energies above their injection energy (grid potential drop $\lessapprox V_a$) plus their initial thermal energy. When the probe is rotated to $\phi = 90^\circ$ so as to collect preferentially electrons perpendicular to $B_0$, an increase in temperature is observed.

Since it is known that fluctuations can alter the time averaged Langmuir probe characteristics, a differential probe has been used to measure $f_\phi(v)$. It consists of two identical one-sided Langmuir probes whose bias differs by a fixed voltage $\Delta V \ll T_e/e$. The difference in the probe currents $I_\phi$ is plotted vs probe voltage yielding a direct display of the distribution function $[\Delta I/\Delta V \propto f_\phi(V)]$. Common mode fluctuation are canceled.

![Fig. 3.](https://via.placeholder.com/150)  
**FIG. 3.** Parallel electron distribution function (normalized to unity) at different end anode voltages. The peak of the distribution is shifted by $v_\phi$ which is smaller than the value calculated from Eq. (2) indicated by the dashed line.

![Fig. 4.](https://via.placeholder.com/150)  
**FIG. 4.** Ion acoustic test waves excited at $\Delta x = \Delta y = \Delta z = 0$ in the presence ($a,b$) and absence ($c,d$) of electron drift. Interferometer traces ($a,c$) show strong damping against $v_\phi$, phase contours ($b,d$) exhibit wavelength asymmetry due to ion drift and wavelength increase due to electron heating with current. $Ar$, $2 \times 10^4$ Torr, $f_{he} \approx 1.3$ MHz.
in this differential measurement. By minimizing the probe size and spacing (2 mm) the effect of all but the shortest wavelength fluctuations is eliminated. The results obtained with the differential probe are in qualitative agreement with those of the single probe, but the magnitude of the temperature increase is found to be smaller (A $T_e/T_\infty \approx 1.7$).

As an independent check of the electron temperature measurements, ion acoustic waves are launched and the sound speed $c_s = (T_e/m_i)^{1/2}$ is derived. Figures 4(a) and (c) show typical interferometer traces in the presence and absence of the axial current flow, respectively. The waves are excited from a "point" source (0.1 mm diam wire, 2 mm length) and propagate both along and across B0, as indicated by the phase front maps of Figs. 4(b) and (d). Without drawing axial current [Figs. 4(c), (d)], the wavelengths are found to differ slightly in both directions along B0, indicating the presence of an axial ion drift from the source toward the end plate. Average and difference values for the two different phase velocities yield the sound speed $c_s = f(\lambda_s + \lambda_i)/2 \approx 1.6 \times 10^5$ cm/sec and the argon ion drift speed $v_{dr} \approx 0.13 c_s$, respectively. When the anode is biased to $V_a = 2.5$, the electron temperature increases from $T_e = 1.1$ eV to $T_e = 2.1$ eV. The ion drift is difficult to establish for $V_a \geq 3$ V since wave propagation opposite to the electron drift is strongly damped.

Finally, the ion temperature has been determined using a retarding-grid ion energy analyzer. In the absence of current flow the ion temperature is presumably below the 0.4 eV resolution of our analyzer, as estimated from sound wave damping data. For $V_a > 4$ V, the ion temperature exceeds $T_i > 0.4$ eV and at $V_a = 10$ V ion tails with energies of 1.1 eV are observed. These values correspond to perpendicular ion energies since the disk-shaped analyzer causes the least perturbation for this measurement. However, the ions are essentially unmagnetized such that $T_i \approx T_e$ (ion cyclotron frequency $f_c \approx 6.5$ kHz, collision frequency $\nu_{ci} \approx 5 \times 10^3$ sec$^{-1}$, cyclotron radius $r_c \approx 3.5$ cm).

IV. TURBULENCE PROPERTIES

The turbulent wave properties can be characterized by two quantities, i.e., the frequency spectrum and the wavenumber spectrum. For a random ensemble of waves the spectra are time-averaged statistical values, and it is generally assumed that time and ensemble averages are interchangeable, i.e., the medium is stationary and homogeneous.

In many experiments the frequency spectrum is the only available data. This partial information is insufficient to identify the type of wave through its dispersion $\omega(k)$. It is certainly an insufficient data basis for identifying a saturation mechanism for the instability. Measured frequency spectra also have to be interpreted with caution since they represent a convolution of the true wave spectrum with the response of the diagnostic tool (probes, scattering setup) which is mostly unknown. Although the frequency spectrum contains valuable information, it has to be supplemented by a measurement of the wavenumber spectrum. The latter involves three-dimensional $k$-vector measurements unless symmetry in one or more dimensions has been established.

A. Frequency spectrum

Typical frequency spectra of the probe current fluctuations $\delta I_p \sim \delta n_e \sim \delta \phi$, where $\delta n_e$, $\delta \phi$ are fluctuations of the electron saturation current, density, plasma potential, respectively) are shown in Fig. 5. At small electron drifts ($c_s < v_e < v_i$) the spectrum exhibits a peak near $\omega/\omega_p \approx 0.7$. With increasing turbulence level the spectrum broadens (0 $\leq \omega \leq \omega_p$) and low frequency components (f $\leq 200$ kHz) assume the largest potential fluctuations. The absolute fluctuation level $\delta \phi$ indicated in Fig. 5 is obtained by comparing the fluctuation level with a calibrated monochromatic signal. Although each Fourier component has a relatively small amplitude, the frequency integrated noise as observed from the waveforms vs time assumes peak-to-peak potential fluctuations of $\varepsilon \delta \phi/T_e \sim 10\%$ or rms values $\varepsilon \delta \phi_{rms}/T_e \approx 11\%$. The rms fluctuation level depends primarily on anode current. For $I_a = 0$, it is approximately $5n_e/T_e \leq 1\%$, at $I_a = 200$ mA it rises to $5\%$, and at $I_a = 300$ mA it is $12\%$. Thus, the normalized wave energy density, $W/T_e = (5n_e/T_e)^2$ varies from $10^4$ to $1.4 \times 10^5$.

The detailed shape of the saturated instability spectrum cannot be readily interpreted. It is observed to depend to some degree on position, probe bias (in particular, below plasma potential), probe size and orientation (in particular, when probe dimensions exceed wavelength). Many of these effects can be understood.
from the complicated three-dimensional wave propagation pattern.

B. Wave vector spectrum

The spatial properties of the turbulent waves are analyzed with two-probe correlation measurements. The fluctuations in the electron saturation currents of two identical small cylindrical Langmuir probes (0.13 mm diam wire, 1 mm long; electron cyclotron radius \( r_{ce} \approx 0.4 \) mm) are applied to narrow band (\( \Delta \omega / \omega \sim 1/50 \)) tuned amplifiers, multiplied and time-averaged. A broadband analog delay line (0 < \( \tau < 10 \) \( \mu \)sec) can be inserted into either channel. The output signal is proportional to the cross power spectral density

\[
H(\Delta \mathbf{r}, \omega) = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} \phi_1(\mathbf{r}, t) \phi_2(\mathbf{r} + \Delta \mathbf{r}, t + \tau) dt. \tag{3}
\]

Here, \( \phi_1 \) and \( \phi_2 \) are the fluctuating signals at the multiplier inputs which are proportional to the Fourier components of the local potential fluctuations detected by probe 1 at \( \mathbf{r} \) and probe 2 at \( \mathbf{r} + \Delta \mathbf{r} \). The signal of probe 2 is delayed by \( \tau \). If the fluctuations are due to propagating waves, one has \( \phi = \phi_0 \cos(\omega t - \mathbf{k} \cdot \mathbf{r}) \) and \( H = \frac{1}{2} \phi_0^2 \cos(\omega t - \mathbf{k} \cdot \mathbf{r}) \). By measuring \( H(\Delta \mathbf{r}, \omega) \) along three coordinates \( \Delta \mathbf{r} = (\Delta x, \Delta y, \Delta z) \) at different values of \( \tau \) one obtains magnitude and direction of \( \mathbf{k} \) at a given \( \omega \), hence the dispersion \( \omega(\mathbf{k}) \). Due to the random variations in phase and amplitude, the magnitude of \( H(\Delta \mathbf{r}, \omega) \) decays for increasing probe separation with characteristic scale length \( \Delta \mathbf{r}_c \), the correlation length. Similarly, there is a finite correlation time \( \tau_c \) at which the magnitude of \( H(\Delta \mathbf{r}, \omega) \) decays by \( 1/e \).

If the fluctuation level shows spatial variations, it is useful to normalize the cross power spectral density so as to obtain a meaningful measure for the correlation length. The normalized cross power spectral density is given by

\[
H_n(\Delta \mathbf{r}, \omega) = \frac{\langle \phi_1(\mathbf{r}, t) \phi_2(\mathbf{r} + \Delta \mathbf{r}, t + \tau) \rangle}{\langle \phi_1(\mathbf{r}, t) \rangle \langle \phi_2(\mathbf{r} + \Delta \mathbf{r}, t + \tau) \rangle}^{1/2}, \tag{4}
\]

where the angular brackets denote time averages. The normalization is carried out by three identical probe scans obtaining, separately, the time averaged terms which are subsequently evaluated by a computer and displayed as topographical maps \( H_n(\Delta \mathbf{r}) \).

Most correlation measurements are performed with a bellow-mounted Langmuir probe which can be scanned over the various planes shown in Fig. 6. Identical reference probes can be positioned at any radial or axial position (\( r, z \)). The dimensions of the exposed probe tips (0.13 mm diam, 1.5 mm length) are, in general, smaller than the measured wavelengths so that the probes should exhibit an isotropic, frequency independent detection pattern. The shielded probe shafts (0.75 mm diam) cause negligible perturbations on plasma and wave properties.

A first estimate of the correlation properties is obtained by displaying the fluctuating probe signals directly on the \( x \) and \( y \) axes of a broadband oscilloscope at equal gain. Perfect correlation would yield a straight line display at an angle of 45°, no correlation generates on the average a circular pattern filled with random traces. As shown in Fig. 7 one observes a rapid loss of correlation when the probes are separated radially [Fig. 7(a)], but long axial correlation lengths along \( B_0 \) [Fig. 7(b)]. Thus, the dominant direction of wave propagation appears to be perpendicular to the electron drift \( v_e \), a result in contrast to the case of unmagnetized plasmas. However, in these correlation measurements, all frequency components are folded together and the results are dominated by the large amplitude potential fluctuations at lower frequencies. More detailed information can be obtained from the cross power spectral density measurements.

A typical trace of the cross power spectral density function in one dimension (\( B_0 \)) is shown in Fig. 8(a). The pattern shows correlated waves, time averaged over about 10⁶ periods, spatially extending over several wavelengths. The observed wavelength is, in general,

\[
\text{FIG. 7. Correlation diagrams of two potential fluctuations } \phi_1, \phi_2 \text{ displayed on the } X \text{ and } Y \text{ axis of an oscilloscope, respectively. As the radial probe separation } \Delta x \text{ in part (a) is increased, the correlation is rapidly lost; in axial direction } \Delta z \text{ in part (b) the correlation length is much longer. } \text{Ar, } 1 \times 10^4 \text{ Torr, } V_x = 5 \text{ V, } B_0 = 140 \text{ G, } 10 \text{ msec exposure time.}
\]
longer than the characteristic wavelength of the eigen-modes by a factor 1/cosa, where a is the unknown angle between the dominant direction of wave propagation and the direction of the probe scan. With probes movable along three orthogonal axes a will be determined uniquely. The spatial decay of the cross power spectral density indicates the loss in phase coherence and/or amplitude decay of the waves intersecting both probes but not necessarily a change in total wave intensity. This is illustrated in Fig. 8(b) by displaying the spectral density functions of both probe signals. The small variations in these traces requires normalization of the cross power spectral density according to Eq. (4) in order to obtain the absolute correlation level and correlation lengths.

Measurements of the cross power spectral density in two dimensions at different frequency components are shown in Figs. 9(a)–(h). These maps give contours of constant phases, i.e., the locations of maxima and minima of the cross power spectral density in the x-y plane across B0 (θ = 90°). The amplitude information will subsequently be given. The phase contours indicate wave propagation at nearly any angle in the perpendicular plane. However, the angular distribution is, in general, not symmetric, i.e., the phase contours are not circular with respect to the reference probe. Noncircular phase fronts can result from asymmetry in the phase velocity and/or asymmetry in the angular probability of wave propagation. These could arise from a combination of nonuniform current distributions, radial wave reflections and wave-wave interactions. The asymmetry, however, varies with frequency and radial plasma and anode properties are highly uniform; no dc radial current gradients are observed. Neither is radial wave reflection important since the correlation length is small compared with the column diameter (12 cm) and the plasma is well separated from the metallic chamber walls (Δr = 9 cm). It is possible that wave-wave interactions lead to preferential directions in k space.

The correlation in the perpendicular plane rapidly vanishes for frequencies $f \gtrsim 1$ MHz. This is not caused by a decrease in probe sensitivity with respect to short wavelength modes because the spectral intensity at either probe does not vanish up to $f \gtrsim f_r = 1.3$ MHz. Rather the direction of optimum correlation changes, which will be shown later.

The amplitude of the cross power spectral density normalized according to Eq. (4) is shown in Fig. 10. The computer drawn data are given either in the form of contours of constant amplitudes [Fig. 10(a)] or as a three-dimensional view in the x-y plane [Fig. 10(b)]. Only one frequency component is shown which is characteristic of, although not identical to, other frequencies. One can see that the amplitude distribution exhibits an even stronger asymmetry around the reference probe located in the center of the plane. In some directions, the correlation length is comparable to the wavelength; in other directions, e.g., along the x axis, the waves are correlated to a high degree (> 50%) over many wavelengths. By translating the measurement plane axially or radially, the inhomogeneities in the cross power spectral density are found to be much larger than those of the spectral intensity measured by a single probe.
No obvious cause for the asymmetries has been found so far, in particular, in view of the fact that at each frequency the pattern looks different. These local asymmetries in wave propagation may not be observable in microwave or laser scattering experiments which average over a large plasma volume, but they have been noticed in related probe correlation measurements.

In order to investigate the third dimension, the probes are scanned axially and radially in a plane orthogonal to that shown before. Figure 11 indicates the phase fronts of the cross power spectral density at $f = 800$ kHz. The phase contours are inclined at a very small angle with respect to $B_0$, resulting in parallel wavelengths $10 < \lambda < 20$ cm with a significant spread in magnitude. Thus, one finds $10^{-2} < k_i / k_0 < 10^{-3}$ but, in general, $k_i / k_0 > (m_e/m_i)^{1/2} = 3.7 \times 10^{-4}$, where $m_e$ and $m_i$ are electron and argon ion mass, respectively. Similar correlation measurements show that in the frequency range $4 \leq 1$ MHz the unstable waves propagate at angles $84^\circ < \theta < 89^\circ$ with respect to the axial magnetic field or electron drift. It has been pointed out previously that probes can perturb the wave properties. The phase jump in Fig. 11 at $\Delta z = 0, \Delta y > 0$ appears to be related to the radial ceramic probe shaft of the stationary reference probe. Although noticeable, the perturbation is localized and does not affect the general shape of the phase contours. Probe effects are minimized by keeping the probe dimensions small compared with the wavelength (the wire diameter of the exposed probe tip is smaller than the Debye length $\lambda_D$ or electron Larmor radius $r_m$) and by operating well above threshold of the instability ($v_a > c_s$) where probe currents have little effect on the large amplitude saturated wave spectrum. The azimuthal asymmetry in the cross power spectral density cannot be related to probe effects since it bears no relation to probe position and motion.
The amplitude of the cross power spectral density in the horizontal y-z plane is shown in Fig. 12. Along the nearly field-aligned phase fronts, the correlation length is long ($\Delta z \approx 25$ cm) for frequencies $f \leq 1$ MHz. Due to the finite column length ($L \approx 80$ cm) low frequency modes may be subject to axial boundaries but these should have little influence on the dominantly radial propagation which again exhibits localized regions of high correlation.

It has been pointed out that the high frequency part of the spectrum ($f \gtrsim 1$ MHz) exhibits different correlation properties from the low frequency part. In Fig. 13 we show the cross power spectral density at $f = 1.2$ MHz vs axial position $z$. One can see good correlation over many short wavelengths along $B_y$ in contrast to the case at lower frequencies. With increasing turbulence level the correlation length, however, decreases. By inserting a time delay $\tau$ in the reference channel and observing the spatial phase shift with $\tau$ one finds that the waves propagate in the electron drift direction, i.e., toward the end anode. The power spectral density of the movable probe shows good homogeneity in this case so that a normalization of the cross power spectral is not necessary. Axial correlations are observed in the frequency range $0.9 < f < 1.4$ MHz. Toward higher frequencies the waves encounter the ion plasma resonance ($\omega_p \approx 1.5$ MHz) where they are strongly Landau-damped ($k \omega_p > 1$), toward lower frequencies the correlation direction turns perpendicular to $B_y$. The transition occurs near the peak of the potential fluctuation spectrum at $f \approx 1$ MHz where, as shown in Fig. 14, two distinct directions of optimum correlation are observed. When the probe is scanned in the axial direction at an angle $\theta = 0^\circ$ with respect to $B_y$, the short wavelength parallel mode is found to be superimposed on a long wavelength mode. As the angle $\theta$ is increased, the long wavelength decreases $\propto \sin^2 \theta$ until at $\theta = 90^\circ$ it is as short as the parallel wavelength for $\theta = 0^\circ$. Similarly, projections of the parallel mode onto inclined planes [see Fig. 6(a)] causes variations in the observed wavelength. It is interesting to note that the correlation optimizes in two preferred directions in $k$ space ($\parallel B_y$, $-\parallel B_y$); a continuous angular distribution of wave vectors would have resulted in wavelengths independent of $\theta$.

With the measurement of the average wave vectors at different frequencies one can plot the dispersion $\omega(k)$ and identify the unstable modes unambiguously. Figure 15 shows a plot of $\omega$ vs $k = (k^2 + \omega_p^2)^{1/2}$ indicating that the lower frequency portion of the dispersion follows a straight line with phase velocity $v_p = \omega/k \approx 1.8 \times 10^3$ cm/sec. This value corresponds to sound speed in argon at $T_s = 1.4$ eV which is somewhat lower than that obtained from test ion acoustic waves ($T_s \approx 2.1$ eV). Since no waves are observed beyond $f \approx 1.4$ MHz ($k \lesssim 1$ mm) which is close to the calculated ion plasma frequency ($\omega_p \approx 1.5$ MHz), the modes are essentially ion acoustic waves. They can propagate for $\omega > \omega_i$, but only in the magnetic field as long as $k_x/k_y > (\omega_i/\omega)^{1/2}$ so that electrons are free to space charge neutralize the ion density perturbations. Although the lower hybrid frequency $\omega_{ih}^2 = (\omega_x \omega_y)^2 + \omega_n^2$ falls into the turbulent spec-

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**FIG. 13.** Cross power spectral density (top trace) along $B_y$ indicating parallel propagation of high frequency modes. The spectral density of either probe (bottom traces) is constant. Reference probe at $\Delta x = 2$ mm, $\Delta y = \Delta z = 0$. $V_x = 2.7$ V, $f_{pi} = 1.5$ MHz.

**FIG. 14.** Cross power spectral densities along a line $\Delta r$ inclined at an angle $\theta$ with respect to $B_y$ [see Fig. 5(h)]. Propagation both along the across $B_y$ can be distinguished from the wavelength projections on $\Delta r$. $T_s \approx 1.2 \times 10^4$ Torr.

**FIG. 15.** Dispersion relation $\omega(k)$ identifying the unstable modes to be ion sound waves. $V_x = 3.5$ V, $T_e = 2$ eV, Ar, $1.2 \times 10^4$ Torr, $f_{pi} = 1.6$ MHz, $\lambda_0 = 0.2$ mm.
FIG. 16. (a) Cross power spectral density vs $\Delta y$ at a fixed delay time $\tau$ in the reference probe channel. (b) Time-position diagram of maxima (circles) and minima (crosses) of the cross power spectral density at $\Delta z = 0$. Waves propagate with $v_p = \Delta y/\Delta \tau = 1.8 \times 10^6$ cm/sec in the $-\Delta y$ direction. (c) At $\Delta z = 10$ cm waves propagate equally in $+\Delta y$ direction. (d) At $\Delta z = 20$ cm waves are excited near $\Delta y = 0$ and propagate away in both directions.

trum ($f_{th} \approx 1.1$ MHz), the parallel propagation and dispersion excludes the existence of unstable lower hybrid waves. On the other hand, ion cyclotron waves may be present in the low frequency part of the spectrum ($f_{ci} \approx 6.5$ KHz) although the ions are effectively demagnetized by scattering from the intense ion sound turbulence. Although the potential fluctuations are large at low frequencies (see Fig. 5), the electric field, $|E| = k_0 b_0$, which determines the particle motion, does not necessarily increase toward lower frequencies.

Measurements of the phase fronts of the cross power spectral density indicate that waves propagate, on the average, along the local normals to the contours. In order to distinguish to which side they propagate, a time delay $\tau$ is inserted into the channel of the fixed reference probe so that the spatial shift of the phase front with increasing $\tau$ indicates the direction of $k(\cdot \Delta r = \omega \tau = \text{const})$. Such measurements show that the high frequency modes propagate parallel to the electron drift. The lower frequency modes ($f < 1$ MHz) are not expected to show preferential directions of wave propagation in the perpendicular plane. Figure 16 gives three examples of position-time diagrams taken under identical conditions at different axial positions. The movable probe scans across $B_0$, the reference probe is fixed near the column axis at $\Delta x = \Delta y = \Delta z = 0$. At $\Delta z = 0$, the cross power spectral density at $f = 700$ KHz for $\tau = 0.6$ usec is shown in Fig. 16(a), and below, in Fig. 16(b), the shift of maxima and minima with $\tau$ is indicated. No amplitude change is observed and the phase shifts continuously with $\tau$, implying that the unstable waves are traveling only in the $-y$ radial direction. At $\Delta z = 10$ cm toward the anode, one finds a different situation, as indicated in Fig. 16(c). With increasing delay, $\tau$, the phase contours remain stationary but the amplitudes vary between maxima and minima. This pattern is characteristic of standing waves or wave propagation in opposite directions with equal amplitudes expected for isotropic wave propagation. At another axial position, $\Delta z = 20$ cm in Fig. 16(c), one can find waves excited near $\Delta y \approx 0$ and then traveling outward in both $\pm \Delta y$ direction. In the directions of purely traveling waves, the correlation lengths are frequently longer than in regions of mixed $k$ vectors. Although at a fixed delay time $\tau$ the cross power spectral density measurement yields only the real or imaginary part of this Fourier transform, the magnitude of the cross power spectral density does exhibit strong azimuthal asymmetries as confirmed by varying the time delay.

V. SUMMARY AND CONCLUSIONS

A detailed analysis of the plasma and wave properties in a current carrying collisionless magnetized plasma has been presented. The relative drift between electrons and ions gives rise to an ion acoustic instability which generates broadband turbulence. In contrast to previous findings, the waves are highly localized within the instability cone in $k$ space; at $\omega / \omega_p \approx 0.7$ propagation is near the marginal instability angle $\theta = \arccos(c_p/c_0) \approx \pi/2$, at $\omega / \omega_p \approx 0.7$ the waves propagate essentially along the drift ($\theta = 0^\circ$). The high frequency modes with lower phase velocities are ion Landau damped for oblique propagation. Azimuthally, the distribution of $k$ vectors is not symmetric. These asymmetries could not be linked to observable inhomogeneities in the plasma parameters nor to boundary effects.

Although the fluctuation spectrum can be elegantly summarized by the cross power spectral density function $S(k, \omega)$, the required spatial Fourier transformation of $H(q, \omega)$ has not been performed because of the complexity of the turbulence spectrum and the problems of displaying a function of four variables. The probe measurement of the cross power spectral density has proved to be a powerful technique for analyzing the $k$-vector spectrum on a statistical basis. This information will be useful in interpreting the electron and ion dynamics in the turbulent plasma to be discussed subsequently.

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