We cannot solve the non-linear equation analytically (why) we can do it numerically for various $e\phi/kT$'s etc

Suppose the potentials are small

Thus $e\phi/kT \ll 1$ so if $\frac{kT_e}{e} \approx 5$ volts $\frac{e}{e}\phi \approx .05$

Then $$e^{e\phi/kT} \cong 1 + \left(\frac{e\phi}{kT}\right) + \frac{1}{2}\left(\frac{e\phi}{kT_e}\right)^2 + \cdots$$

Then $\left(e^{e\phi/kT_e} - 1\right)[en_0]$ becomes $\Rightarrow en_0\left[1 + \frac{e\phi}{kT_e} - 1\right]$

and we now have

$$\epsilon_0 \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial \phi}{\partial r}\right) \cong \frac{e^2 \phi n_0}{kT_e} \qquad e\phi \ll kT_e$$

The solution of this equation is:

$$\phi(r) = \frac{A e^{-r/\lambda_D}}{r} \qquad \lambda_D = \left(\frac{kT\epsilon_0}{ne^2}\right)^{1/2}$$

prove it:

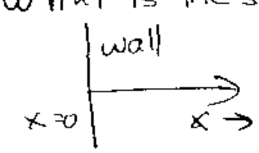
$$\frac{\partial \phi}{\partial r} = -\frac{1}{\lambda_D}\frac{Ae^{-r/\lambda_D}}{r} - \frac{1}{r^2} A e^{-r/\lambda_D}$$

$$r^2 \frac{d\phi}{\partial r} = -\frac{r}{\lambda_D} A e^{-r/\lambda_D} + A e^{-r/\lambda_D}(-1)$$

$$\frac{\partial}{\partial r}\left(r^2 \frac{\partial \phi}{\partial r}\right) = -\frac{1}{\lambda_D} A e^{-r/\lambda_D} + \frac{+r}{\lambda_D^2} A e^{-r/\lambda_D} + \frac{1}{\lambda_D} A e^{-r/\lambda_D}$$

$$\frac{\epsilon_0}{\lambda_D} A e^{-r/\lambda_D} = \frac{ne^2}{kT_e} A e^{-r/\lambda_D}$$

$$\text{or} \quad \lambda_D = \frac{kT_e \epsilon_0}{ne^2}$$

QUESTION what is the solution if a slab is put in the plasma and 

?

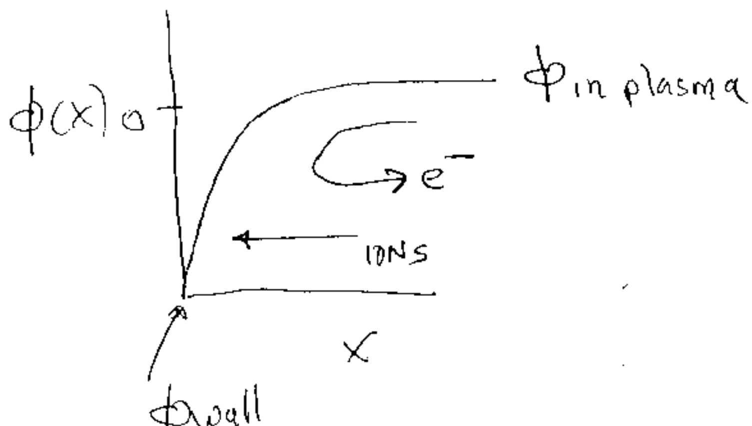
What happens near a wall

Suppose $T_{ION} = 0$ or $T_e \gg T_{ION}$ (temperatures)

remember $T_{ION} = 0$ was assumption we made

Since electrons are moving faster they strike the wall and the wall charges up negatively

Then additional electrons will be repelled when the wall becomes negative enough

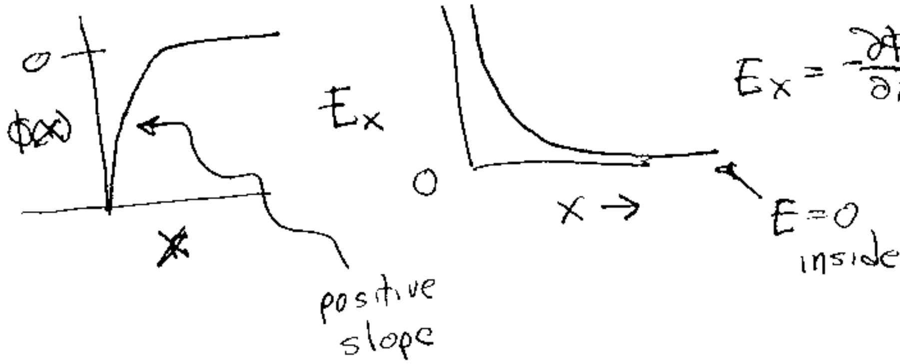


Thus there is always an electric field between a plasma and a wall $\vec{E} = -\nabla\phi$ or in 1D $E_x = -\frac{\partial\phi(x)}{\partial x}$

NOTE if the wall is negative ions will rain down on it even if $T_i = 0$. This is because directed motion

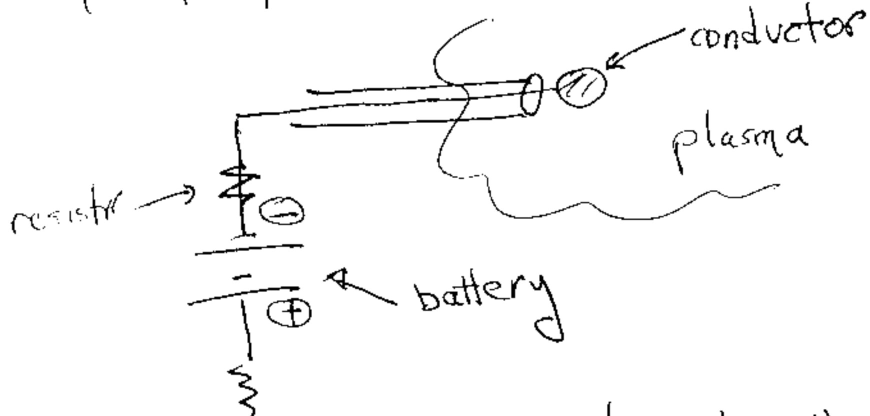
$\frac{1}{2} M_{ION} V_x^2$ = kinetic energy is <u>not</u> temperature

(13) So since



$$E_x = -\frac{\partial \phi}{\partial x}$$

What happens when you put a small metal conductor in a plasma and then apply a bias voltage to it



If the disc is negative then it will draw ions to it (note far-away it will be shielded out)

$V = IR$ ohm's law. (Question what is ohms law in a plasma? Is it this (answer - No))

But $I = \int_A \vec{J} \cdot \hat{n}\, da$

current density amp/cm²

$\vec{J} \equiv n e \vec{V}_{drift}$

#/cm² — charge — cm/sec

(17) For every ion that is collected an electron must be lost somewhere to maintain charge neutrality

Suppose the plasma is Maxwellian with a temperature T_e (electrons) and T_i for ions.

in 1 Dimen $\quad \frac{1}{2} m_e v_e^2 = kT_e \qquad \frac{1}{2} M_I v_i^2 = kT_i$

Boltzman's constant

So if $T_e = T_i$ $\quad V_e \gg V_i$

Note $\quad V_e = 4.4 \times 10^7 \sqrt{T_e} \qquad V_i = 9.8 \times 10^5 \sqrt{\frac{T_i}{\mu}}$

T in electron volts $\quad \mu = \frac{\text{Mass}}{M_{\text{hydrogen}}}$

Therefore $\quad J_{ion} \ll J_{electron}$

$$\frac{J_{ion}}{J_{elec}} = \frac{9.8 \times 10^5}{4.4 \times 10^7} \frac{1}{\mu^{1/2}} \sqrt{\frac{T_i}{T_e}} = .022 \frac{1}{\mu^{1/2}} \sqrt{\frac{T_i}{T_e}}$$

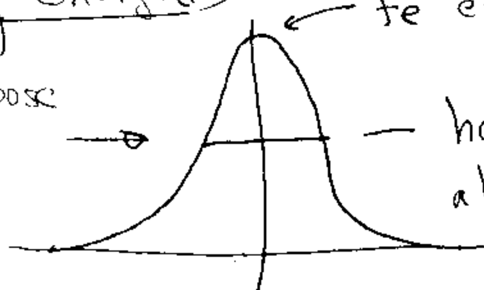
So for hydrogen if $T_i = T_e$ $\quad J_{ion} = 2\% \, J_{electron}$

Thus if you make the probe (+) you will get 50x the current in this case.

Suppose you do not bias the probe at all. More electrons per unit time will hit it than ions and it will become negatively charged and collect electrons

f_e electron distribution function

Now suppose you want to collect no current



half width $\approx \frac{1}{2} kT_e$ at half max

If the probe charges to $\frac{1}{2} kT_e$ it will repel

(15)

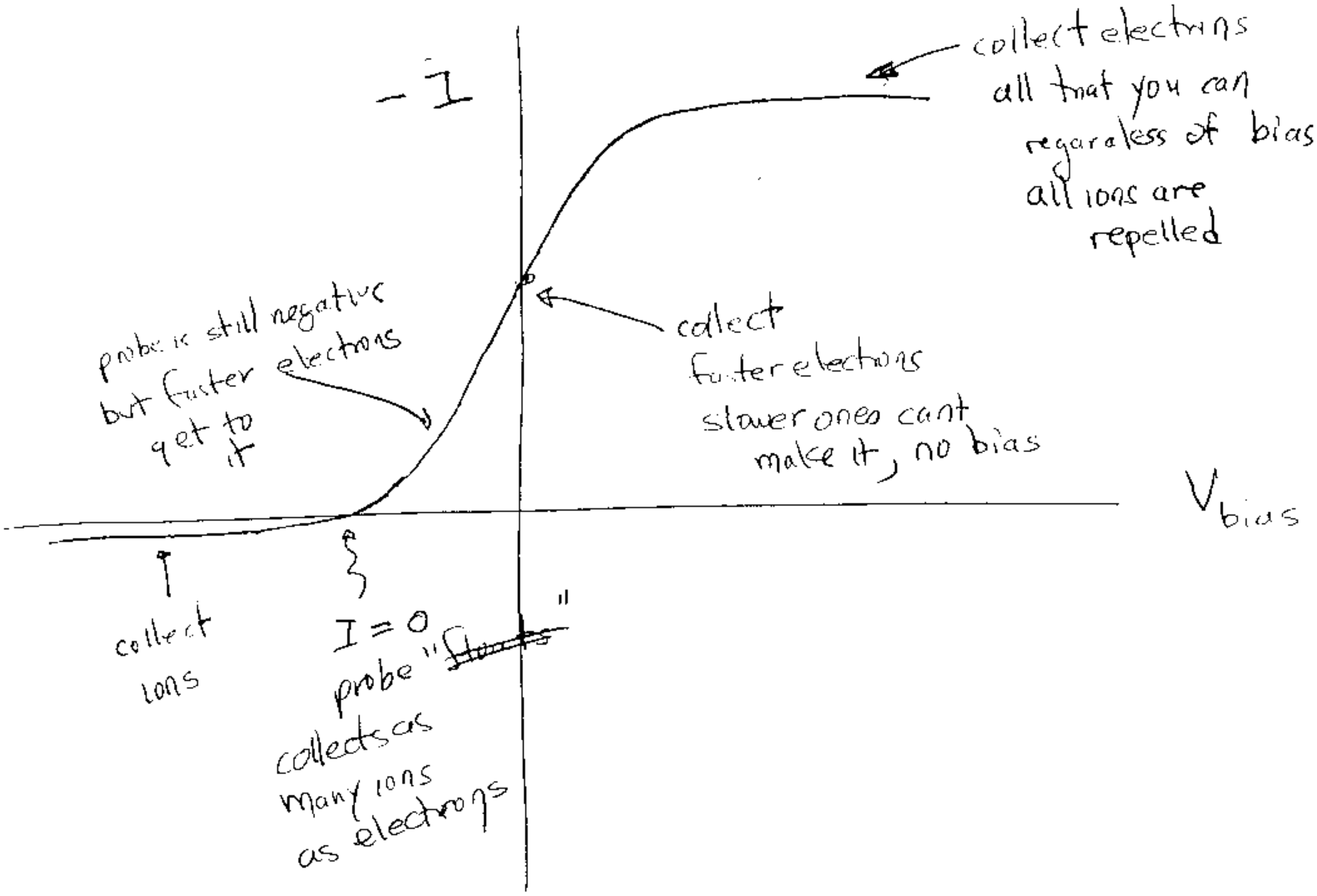
repel  all the electrons under the shaded part. The rest will hit

But after it "floats" in the plasma it can draw no net current so the number of electrons hitting it must equal the number of ions. Since the ion current is ≈ 2% then it must charge up negatively to repel 98% of electrons which means $e\phi \approx (2-3)\, kT_e$

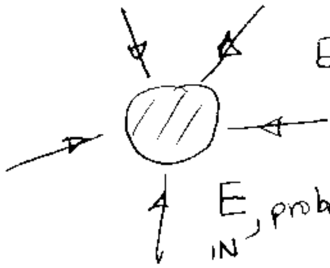
$$e^{-3} \approx .05 \quad \text{and} \quad \phi(x) \approx e^{-e\phi/kT}$$

electron charge

Suppose we now vary the bias on the probe and collect the current. We get

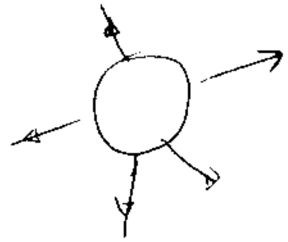


So if

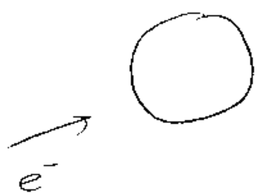


E extends out only a few λ_D

E_{IN}, probe is positive collect electrons

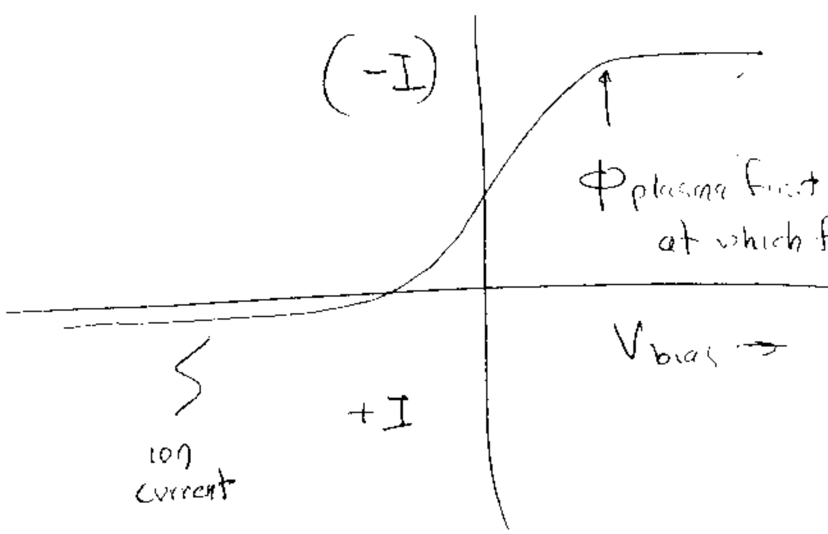


E_{out}
probe is negative
collect IONS



$\vec{E} = 0$ collect all electrons freely flow to the probe
thus $\Phi_{probe} = \Phi_{plasma}$

The probe is thus negative and repels all ions So this happens



Φ_{plasma} first bias voltage
at which full electron current is collected

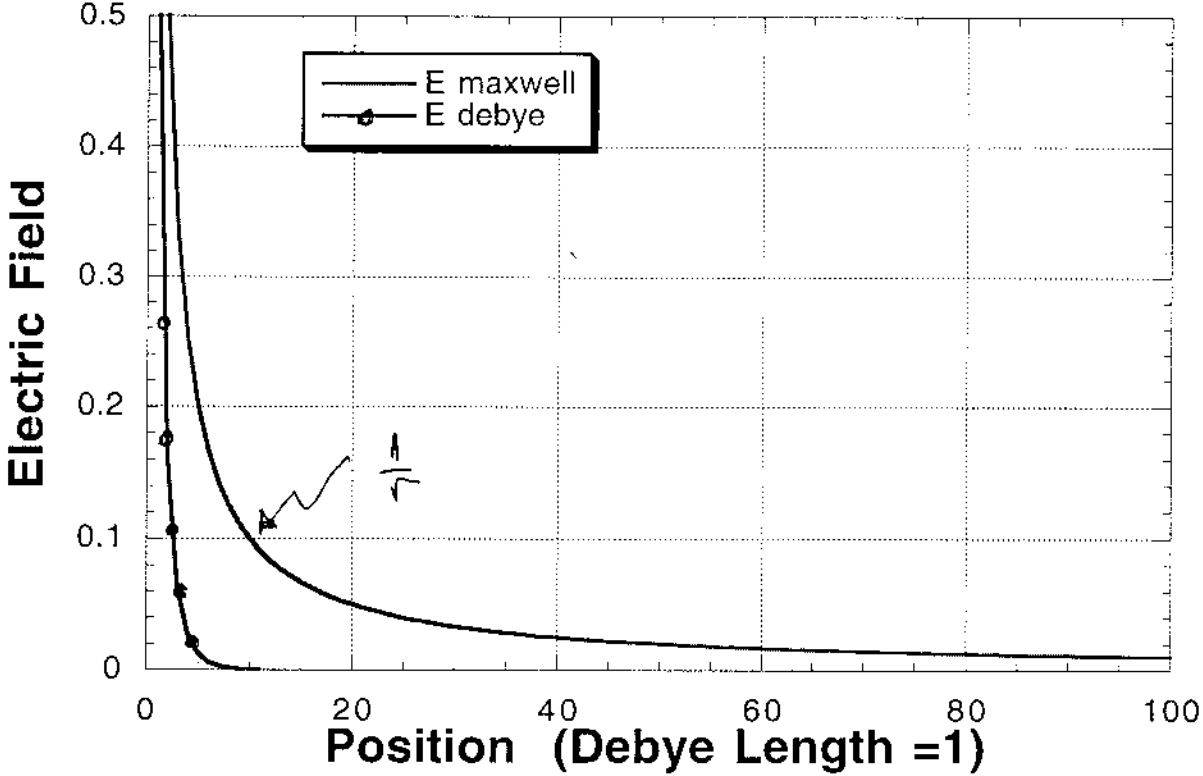
There are formulas for the ion and electron currents which come from solving the Debye type equation. Rather than go into detail

$$I_{ION} \cong ne\left(\frac{kTe}{m_I}\right)^{1/2} A$$

note depends on Te not Ti !

$$I_{electron} = J_{electron} A = -n e v_e A$$

probe area



E maxwell
E debye
$\frac{1}{r}$
Electric Field
0.5
0.4
0.3
0.2
0.1
0
0
20
40
60
80
100
Position (Debye Length =1)