LAPTAG

Lecture notes on Waves/Spectra Noise, Correlations and ….

References: Random Data 3rd Ed, Bendat and Piersol,Wiley Interscience

Beall, Kim and Powers, J. Appl. Phys, **53**, 3923 (1982)

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Lecture 6, April 27, 2004

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Amazing Fact !

The autocorrelation function and spectra are related to each other by

The Fourier Transform!!

Spectra of x(t)x(t+rΔt) = Fourier Transform of the Autocorrelation function

Formally

$$S_{xx}(f) = \int_{-\infty}^{\infty} R_{xx}(\tau) e^{-i2\pi f \tau} d\tau$$

This was proven by Von Neuman and is part of the mathematics of cybernetics or

Information. This was essential for the development of computers

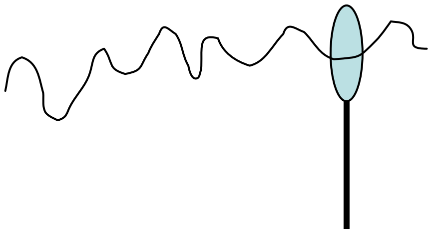
Cross Spectral Functions

Huh ?

Suppose you have many waves which start from point a

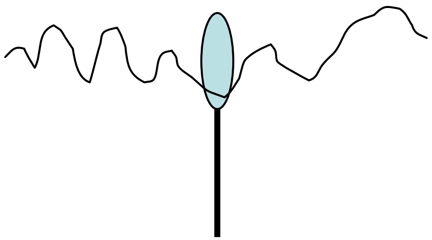
The waves change as they move !

a



b

They pass a probe at point b which detects them



c

They pass another probe at point c which detects them

How well are the waves which pass point a correlated with the waves that pass point b ?

The waves can decay as they move, or grow. The frequencies of the waves that make up the signal can change as they move as well.

1) The Cross correlation function tells us how strongly the signal at probe a is correlated (or related) with that at probe b.
2) The Cross Spectral Function tells us how the signals to the probes are related frequency by frequency
3) There is (as we can guess) a relation between these functions and FFT’s

The Cross Correlation function is similar to the autocorrelation except now we have two signals x(t) and y(t). These are the signals from the two probes respectively.

R_{xx} is the cross correlation function

$$\widehat{R}_{xy}(r\Delta t) = \frac{1}{N-r}\sum_{n=1}^{N-r} x_n y_{n+r} \qquad r = 0,1,2,3,\ldots m \text{ with } m<N$$

x is the digital data at probe a so $x_n = x(t_0 + n\Delta t)$

y is the digital data at probe b $y_n = y(t_0+[n+r]\Delta t)$

Lets think about what this means. There are a total of N time steps in the data. N = 1,2,3,4…N. For each value of r we have to sum the product of x and y over all the n's. This gives us one value for R. W then have to do the sum/product N-r times.

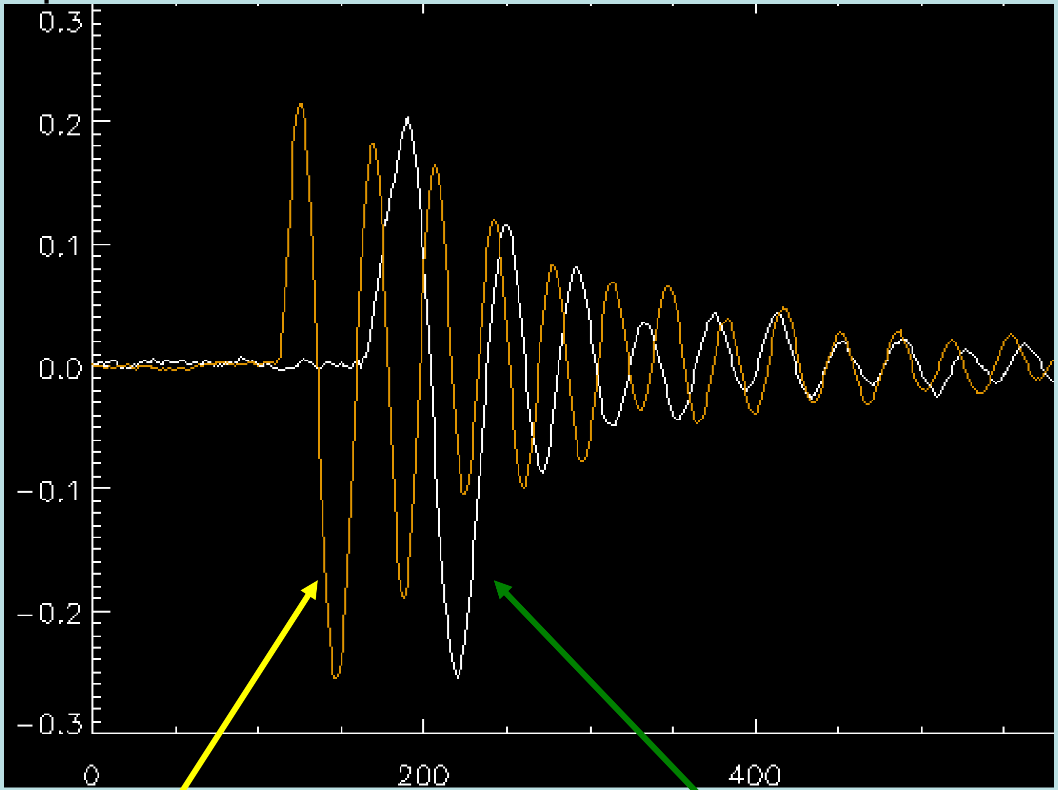
Amazing fact 2

$$\tilde{R}_{xy} = \tilde{B}_x \tilde{B}_y^*$$

(complex) cross spectral density

y component of complex Fourier transform of B

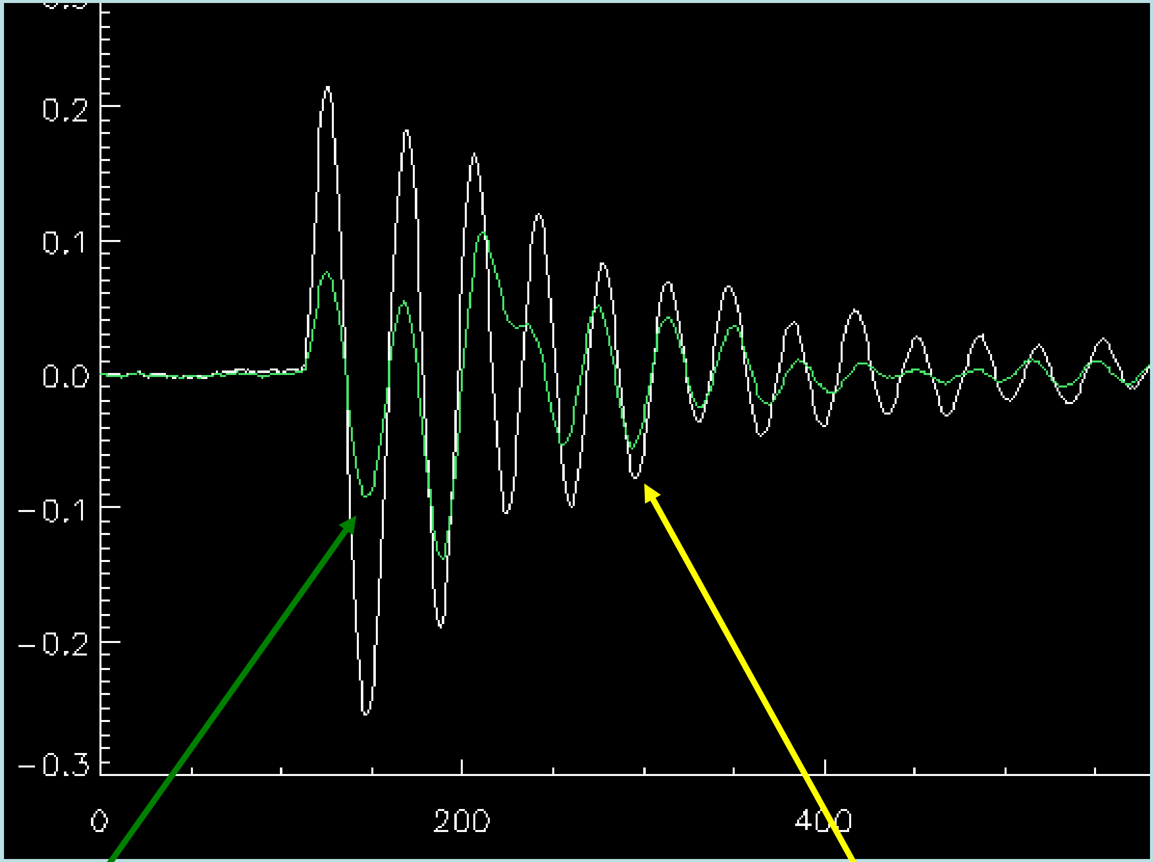
Here is the example of one component (B_y)of the magnetic field of a wave (Alfven wave) acquired in an experiment on the LAPD device at UCLA. The data was acquired as a function of time (each $\Delta t = 0.01\ \mu sec$) at the same (x,y) location three meters apart.



By at first data plane

By at second plane 2 meters away

What does the cross correlation look like?



200 = 2 μs

green = cross correlation

white = By on closest plane

The signals are well correlated for about 3 μs !

What about the dispersion relation of the waves? Suppose we have many frequencies in the noise , not like the case just displayed. We want to find out what

$$\omega/k = ?.$$

For light in vacuum things are simple $\frac{\omega}{k} = c$ where c = $3X10^8 \frac{m}{\sec}$ in general

In general $\frac{\omega}{k} =$ f(ω) here the phase velocity is a complicated function of ω

$\omega = 2\pi f$ and we can get f from the spectra (Fourier Transform)

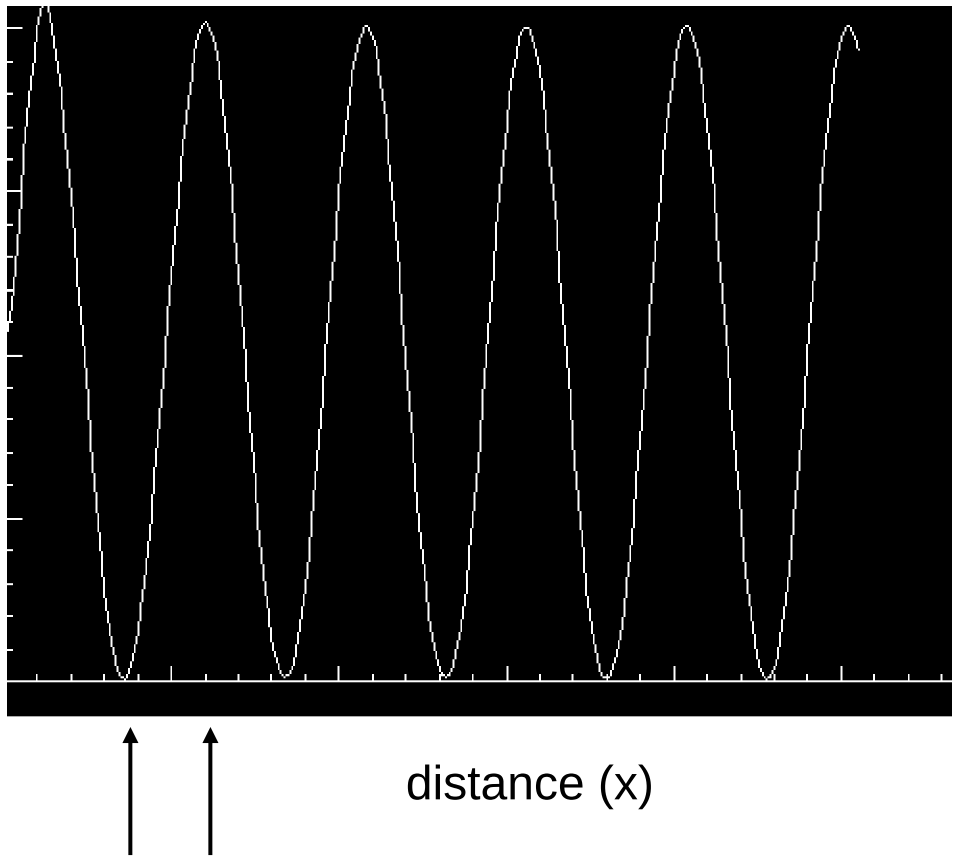
What about k ?

Remember k is the wavenumber where $k = 2\pi/\lambda$

How do we use mathematical techniques to get k from the data ?

Note there are is a spectrum of frequencies in the data so there will also be a spectrum of k

We must measure the wavelength* by moving one probe with respect to the other. If we move a probe a distance such that our received signal is 180 degrees out of Phase from where we taken our first measurement we have moved by one wavelength.



distance (x)

Distance so phase changes by π; we travelled ½ λ

* Strictly speaking the wave could move in any direction so we have to find its wavelength in all three directions. For now lets discuss wave moving along the x axis only!

1) The wavelength is related to phase changes

2) If we have two probes in the plasma separated by a distance δx, and they are both simultaneously recording wave data then

the phase change between them is:

↓

3) $$\delta\theta = k\,\delta x$$ k is the wavenumber

Now we have many, many waves going all superimposed and each time we record data from the two probes it looks different (because the waves are random)

How do we use this data to find the dispersion relation?

We want the dispersion relation because it tells us which waves they are!

How to do it

1) We have recorded n experiments (repetition of the same experiment) each with r time steps from two probes. The probes record the signals at time intervals Δt.
2) Call the signal from the first probe $B_1(r_1\Delta t)$ and that from the second probe $B_2(r_2\Delta t)$
3) Since we have many frequencies in the recorded data we first do an FFT of both signals to get:

$$B_1(x,\omega) = \frac{1}{N}\sum_{l=1}^{N} B_1(x, l\Delta t)\, e^{-i\omega l\Delta t}$$

Now the magnetic field has a real and an imaginary component and is frequency space There is a similar expression for B_2

Now that we have Fourier Transformed each B field we now use them to find the cross spectral function H.

Position of probe B_2

$$\hat{\tilde{H}}(\chi,\omega)=\frac{1}{M}\sum_{j=1}^{M}B_1^*\left(x_{1j},\omega\right)B_2\left(x_{2j},\omega\right)$$

$\chi_p = x_2 - x_1$

(probe separation)

M = number of events in which B_1 and B_2 were simultaneously stored

Complex conjugate of Fourier transform of B_1

$$\hat{\tilde{H}}\left(\chi_p,\omega\right)=\frac{1}{M}\sum_{j=1}^{M}B_{1j}^{*}\left(x_1,\omega\right)B_{2j}\left(x_2,\omega\right)$$

Note H is a complex function since it comes from Fourier Transforms which give us complex functions

$$\hat{\tilde{H}}\left(\chi_p,\omega\right)=C\left(\omega\right)+iQ\left(\omega\right)$$

Real Part of H

Imaginary Part of H

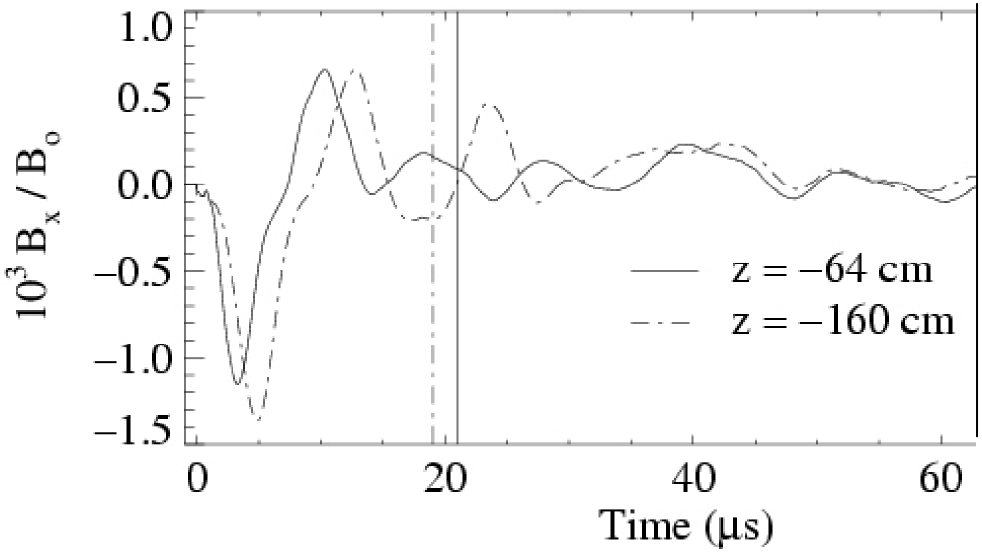
The local wavenumber is :

$$\hat{k}(\omega) = \frac{1}{\chi_p} \tan^{-1}\left[\frac{Q(\omega)}{P(\omega)}\right]$$

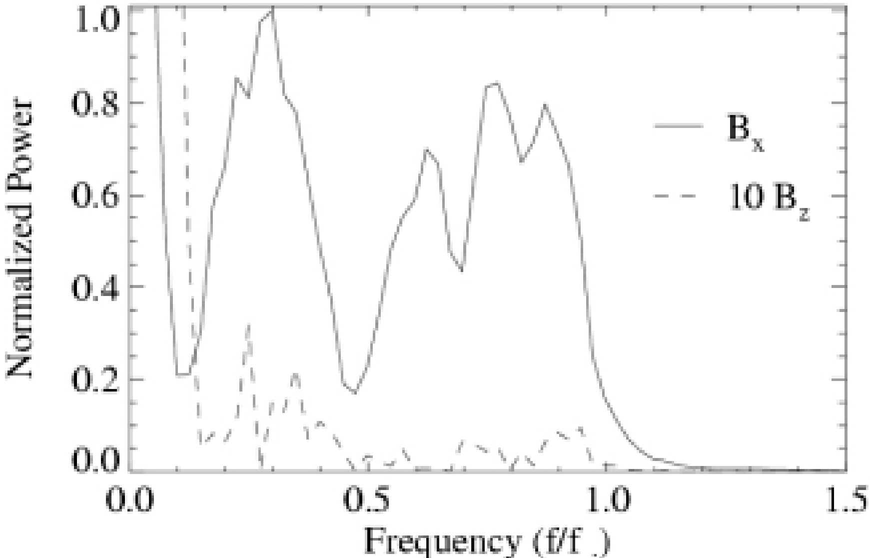
Probe separation

Real and Imaginary Parts of the Cross Spectral Function

An example:

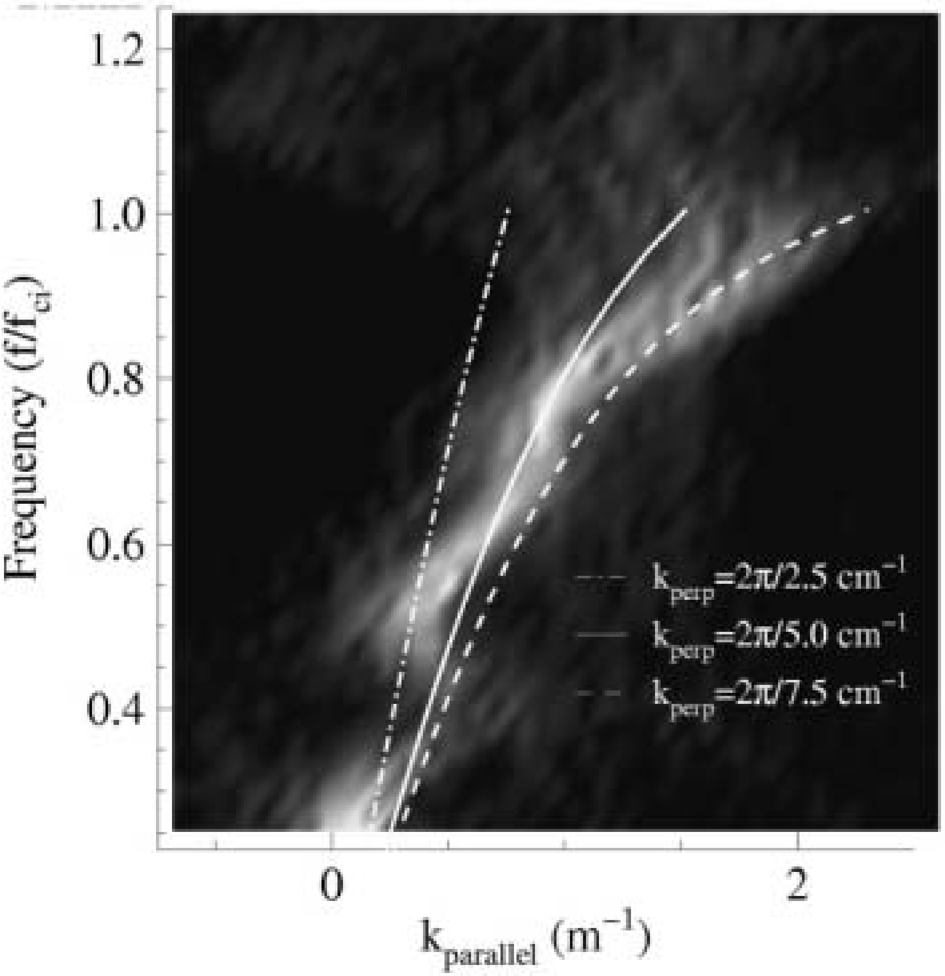


Two probes 96 cm apart, measurement of Bx



Spectra of one of the signals

Phys. Plasmas, M. Van Zeeland, W. Gekelman, S. Vincena, J.Maggs, 10, 1243 (2003)



Suspected Wave

(Kinetic Alfvén Wave)

$$\frac{\omega}{k_{\parallel}} = \left[1 - \left(\frac{\omega}{\omega_{ci}}\right)^2 + \left(\frac{k_{\perp}}{\rho_{cs}}\right)^2\right]^{1/2} V_A$$

$$V_A = \sqrt{\frac{B^2}{\mu_0 n M_I}} \approx 5X10^7 \text{ cm/s} \qquad \omega_{\text{ci}} = \frac{qB}{M_I} \approx 7.2X10^5$$

$$\rho_s = \frac{c_s}{\omega_{ci}} \approx 7mm$$

ion cyclotron frequency

ion acoustic wave speed

$$\hat{k}(\omega) = \frac{1}{\chi_p}\tan^{-1}\left[\frac{Q(\omega)}{P(\omega)}\right]$$

The Power Spectra Come from the Fourier Transforms of each Probe. We investigated this in previous lectures

$$\widehat{S}_1(\omega) = \frac{1}{M}\sum_{j=1}^{M} B^{*}{}_{1j}\left(x_1,\omega\right) B_{1j}\left(x_1,\omega\right)$$

$$\widehat{S}_2(\omega) = \frac{1}{M}\sum_{j=1}^{M} B^{*}{}_{2j}\left(x_2,\omega\right) B_{2j}\left(x_1,\omega\right)$$

$\wedge$

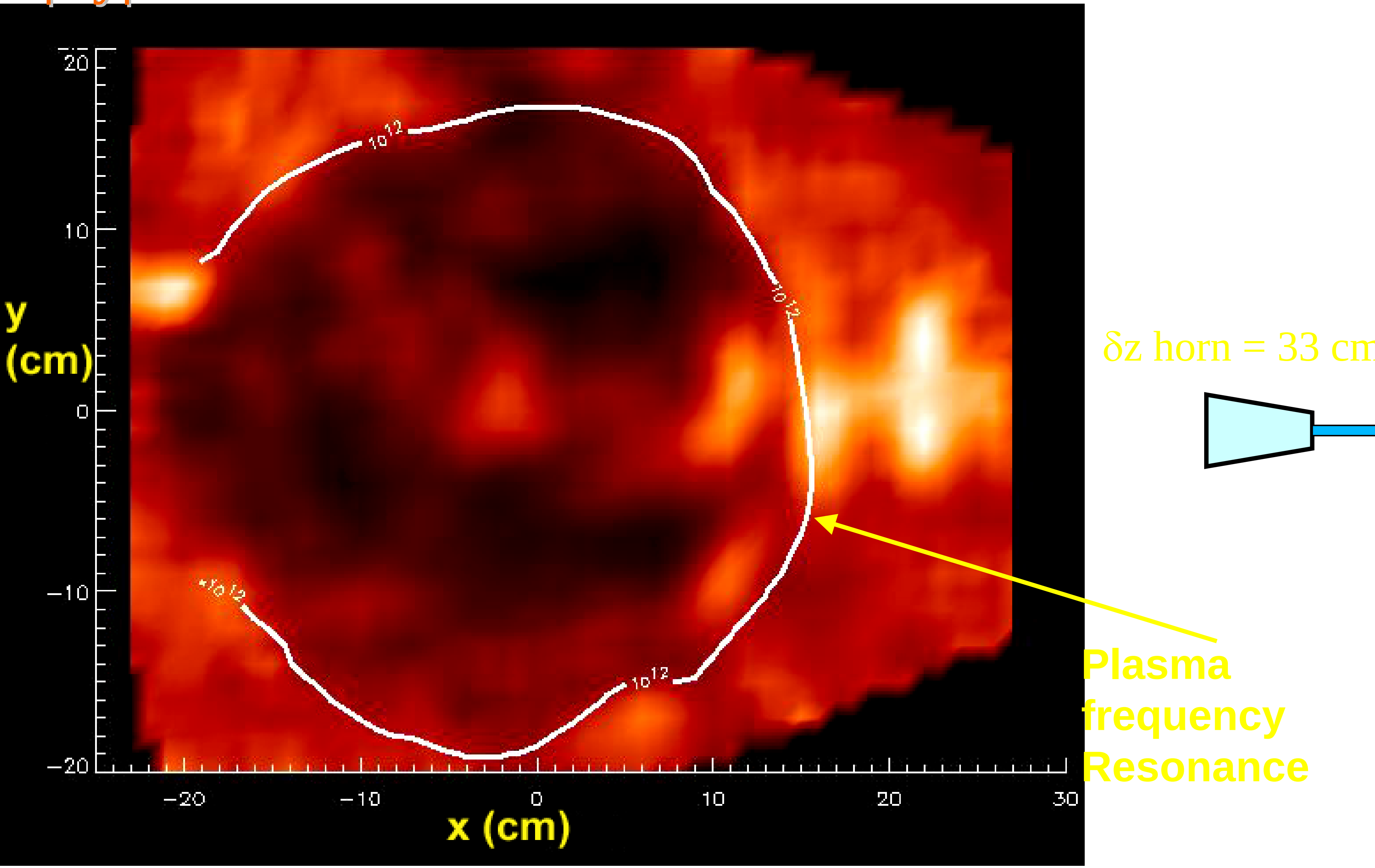
symbol
denotes
average

The Coherency Spectrum is:

$$\gamma(\omega) = \frac{\left|\hat{\tilde{H}}(\chi, \omega)\right|}{\sqrt{\hat{S}_1(\omega)\hat{S}_2(\omega)}} \qquad \chi = x_2 - x_1$$

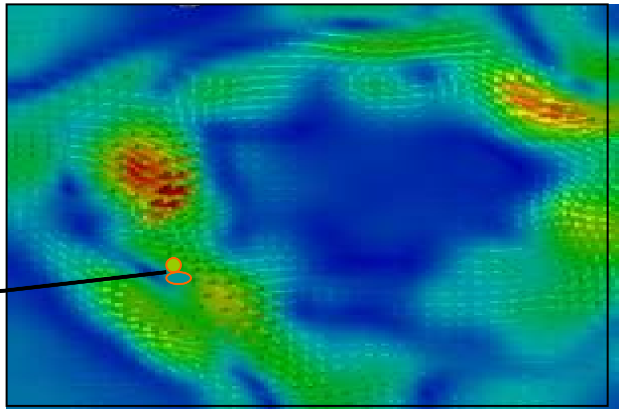
Denominator guarantees that $\gamma(\omega)$ is between zero and one!

|Ey| incident 'O' mode microwaves



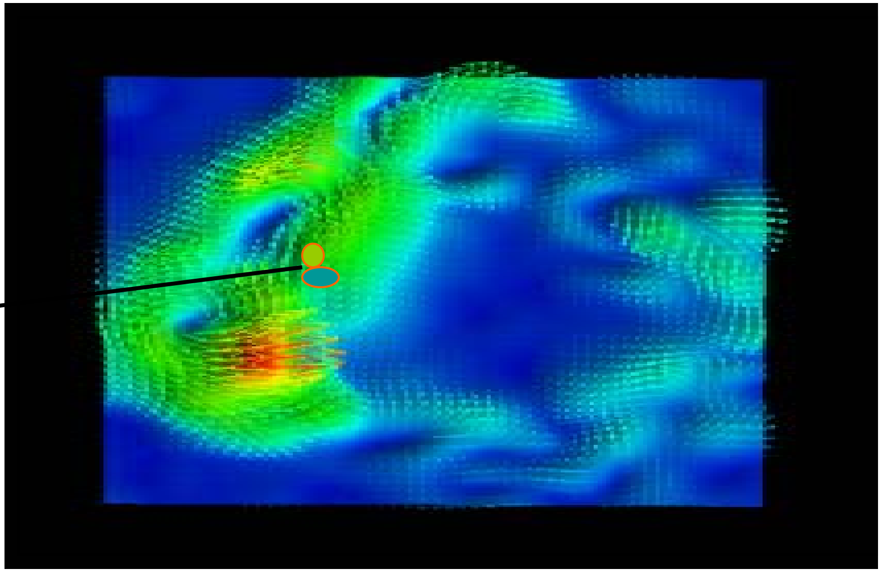
Correlations

(400 positions)*(3 componets)*(6000 timesteps)*(50 shots)=$3.6X10^8$ numbers



Fixed Probe

B_0 **Machine axis**



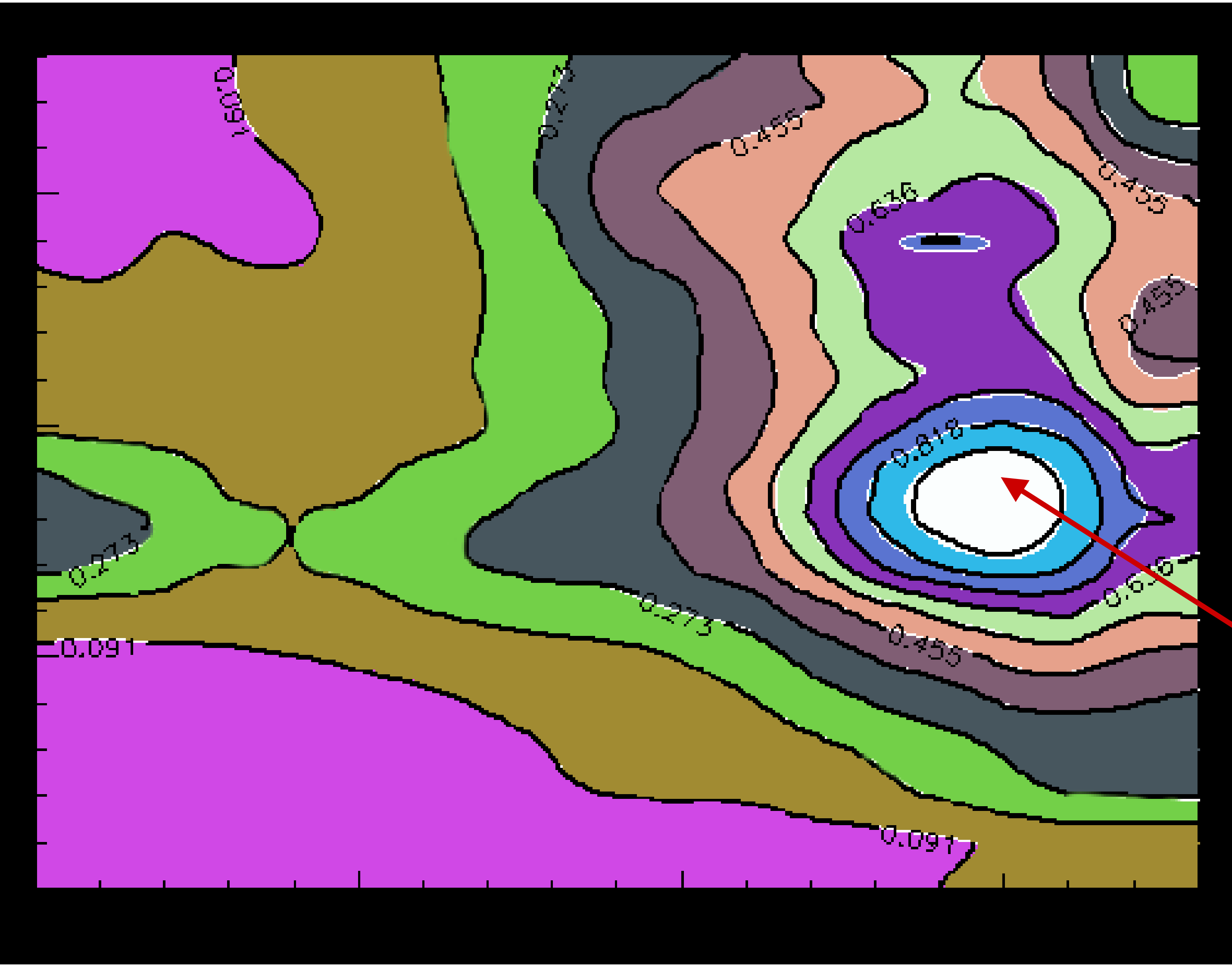
Movable Probe

(computer controlled)

Coherency

$$\gamma_{Byf-Bxm} = \frac{|<\tilde{B}_{ym}\tilde{B}^{*}_{xf}>|}{|<\tilde{B}_{ym}>||<\tilde{B}_{xf}>|}$$

From 2 plane correlation measurement



Ave phase

$$\tan(\theta) = \frac{\mathrm{Im}\left(<\tilde{B}_{ym}\tilde{B}^{*}_{xf}>\right)}{\mathrm{Re}\left(<\tilde{B}_{ym}\tilde{B}^{*}_{xf}>\right)}$$

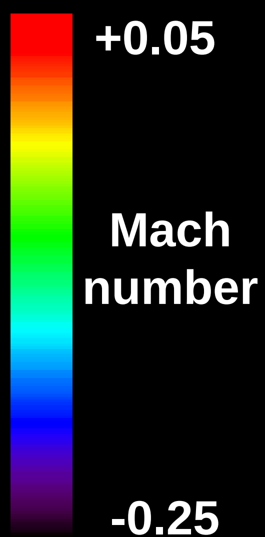
θ = 2.55 radians

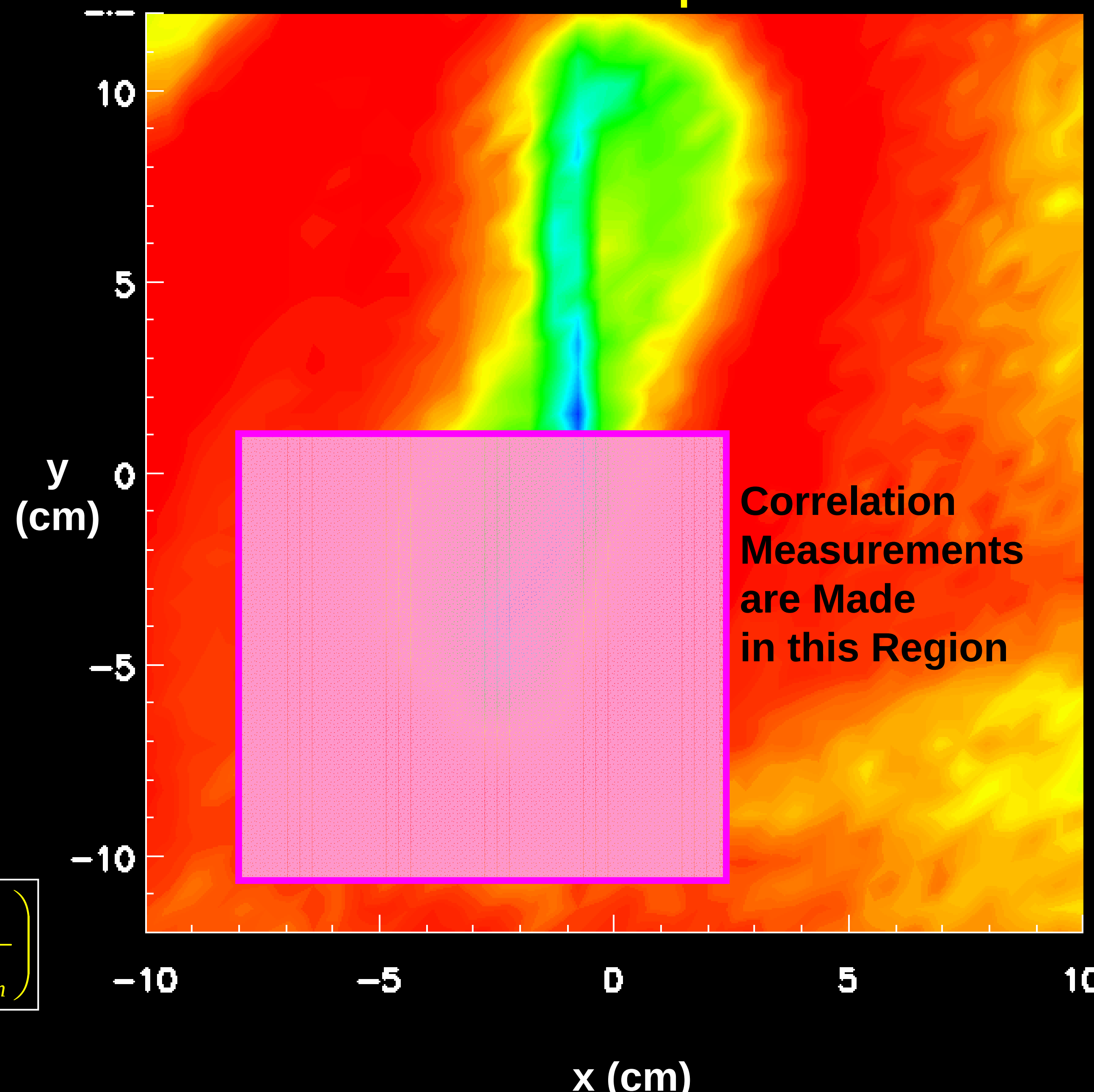
δz = 66 cm

T=1000μsec

Parallel Ion Flow in a Perpendicular Plane

Time during spontaneous fluctuations





$$M_{\parallel} = \frac{1}{2}\ln\left(\frac{I_{Sat-Upstrem}}{I_{Sat-Downstream}}\right)$$

Density Fluctuations Due to Drift Waves

Frequency:
0.2Fci