Plasma Physics

- Space Physics... 99% or more of the Universe is in the plasma state
- Solar System Physics: magnetosphere, solar wind, magnetotail, auroral regions
- Astrophysical Plasmas: galactic jets, cosmic ray production, accretion discs, solar flares
- Shuttle and near Earth Physics: Plasma Propulsion, spacecraft charging
- Thermonuclear Fusion
- Industrial Plasma Processing: coatings ion deposition
- Plasma Chemistry: new compounds
- Environmental Physics: Plasma Torch
- Lasers: Ion Argon Laser, Excimer Laser...
- Lightning (one per second on the Earth, some go up to 80 km)
- Lighting: (florescent bulbs, gas discharges, advertising, plasma display lamps
- Fast Switches: ignitrons, thyratrons, opening switches
- Pure Ion Plasmas: transition from collection of single particles to crystals
- Military applications: gyratrons, free electron laser, microwave sources
- Hydrogen Bomb
- Future Particle Accelerators: Beat Wave Accelerator, Wakefield Accelerator
- Basic Understanding of highly nonlinear systems

Plasma Formulas and Useful Relationships. Mostly taken from the NRL Plasma Formulary. T is in e.V., μ is units if proton mass, B is in Gauss, Z is the ionization state, γ the adiabatic index, M is ion mass , m is electron mass. length is in cm, velocity in cm/sec

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Frequencies:

Electron Gyrofrequency : $\omega_{ce} = \dfrac{eB}{m} f_{ce} = \dfrac{\omega_{ce}}{2\pi} = 2.8 * 10^6 \, Mhz / Gauss$

Ion Gyrofrequency: $f_{ci} = 1.52 \dfrac{B}{\mu} * 10^2 Z \qquad \omega_{ci} = \dfrac{eB}{M}$

Electron Plasma frequency: $f_{pe} = 5.64 * 10^4 \sqrt{n_e} \qquad \omega_{pe} = \sqrt{\dfrac{4\pi n e^2}{m}}$

Ion Plasma Frequency: $f_{p_i} = 2.10 * 10^2 Z \sqrt{\dfrac{n}{\mu}}$

Lengths:

Debye Length $\lambda_D = \sqrt{\dfrac{kT}{4\pi n e^2}} = 7.43 * 10^2 \sqrt{\dfrac{T}{n}}$

Plasma Skin Depth $\delta = \dfrac{c}{\omega_{pe}} = 5.31 * 10^6 \dfrac{1}{)n}$

Ion Gyroradius $R_{ci} = \dfrac{v_{thi}}{w_{ci}} = 102 \dfrac{\sqrt{\mu T_i}}{ZB}$

Electron Gyroradius $R_{ce} = \dfrac{v_{the}}{\omega_{ce}} = 2.38 \dfrac{\sqrt{T_e}}{B}$

Velocities:

Electron Thermal Velocitiy $v_{the} = \sqrt{\dfrac{kT_e}{m_e}} = 4.2 * 10^7 \sqrt{T_e}$

Ion Thermal Velocity $v_{thi} = \sqrt{\dfrac{kT_i}{M_i}} = 9.79 * 10^5 \sqrt{\dfrac{T_i}{\mu}}$

Ion Sound speed $$c_s = \sqrt{\frac{\gamma Z k T_e}{M_i}} = 9.79 * 10^5 \sqrt{\frac{\gamma Z T_e}{\mu}}$$

Alfvén speed $$V_A = \frac{B}{\sqrt{4\pi n M}} = 2.18 * 10^{11} \frac{B}{\sqrt{\mu n}}$$

Other useful numbers:

10^{-6} Torr = $3.54\text{X}10^{10}$ particles /cm3

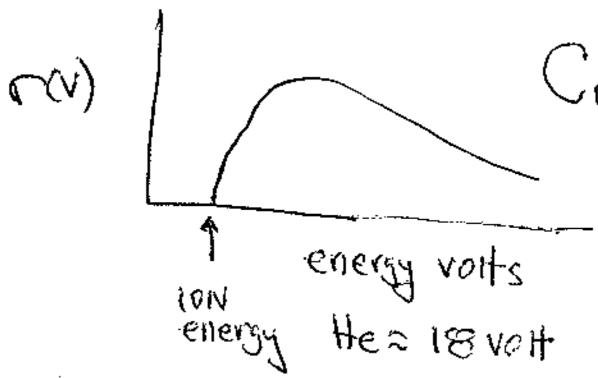
1 joule = 0.24 cal = $6.2\text{X}10^{18}$ e.V.

1 e.V. = 11.600 degrees (equivalent temp)

Laptag Plasma Physics

(I) Plasma is ionized matter

ionization energy → how many volts it takes to remove an electron



Cross section for ionization

He ≈ 18 volt

1 volt ⇒ 11,800°C temperature of a 1eV plasma.

(II) Ions and Electrons need not be at the same temperature

(IIa) Plasmas are often NOT in thermal equilibrium

A plasma may have some neutral particles in it, if it is not very hot

ie Flourescent light $T_e \approx 2eV$ T_i = room temperature and 99.9% neutrals at room temp.

Tokomak $T_i = 30$ keV = 348 million degrees
$T_e = 20$ keV
% ionization = 100%

When does a collection of charged particles become a plasma?

Each particle "feels" a force due to electric and magnetic fields (we totally neglect gravity in most cases)

(1) $F_i = m \frac{dv_i}{dt} = q_i (\vec{E} + \vec{V_i} \times \vec{B})$ $i =$ i^{th} particle

$\vec{E}$ and $\vec{B}$ are the electric and magnetic fields due to:

(2) $\vec{B} = \vec{B}_{\substack{imposed \\ external}} + \vec{B}_{\substack{plasma \\ particle \\ current}}$

(3) $\vec{E} = \vec{E}_{imposed} + \sum_i E_i$

How many particles let $i \approx n\, VoL = 10^{10}/cm^3\ \pi (15)^2 60$

n = number density = number/volume

15cm radius

$\approx$ length

for LAPTAG chamber

$i = 4.2 \times 10^{14}$ particles

This is the number of simultaneous differential equations to be solved. This is impractical

Also $\vec{E}, \vec{B}$ are related by Maxwells equations (in mks)

Gauss LAW (4) $\oint \vec{E} \cdot \hat{n}\, dA = q/\epsilon_0$ or $\nabla \cdot \vec{E} = \rho/\epsilon_0$ ← charge/volume

closed surface (5) $\oint B \cdot \hat{n}\, dA = 0$ $\nabla \cdot \vec{B} = 0$

No monopoles

$\epsilon_0 = 8.85 \times 10^{-12}$

(6) $\int \vec{E}\cdot d\vec{l} = -\frac{\partial}{\partial t}\int \vec{B}\cdot\hat{n}\,dA \Rightarrow \nabla\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}$

induced electric field and changing flux

and

(7) $\underbrace{\frac{1}{\mu_0}\int B\cdot dl = i}_{\text{ampere' Law}} + \epsilon_0\int E\cdot\hat{n}\,da \Rightarrow \nabla\times\vec{H} = \vec{J} + \frac{\partial\vec{D}}{\partial t}$

Maxwells' Term

electric susceptability in a plasma not ϵ_0

with $\vec{B} = \mu\vec{H}$ $\quad \vec{D} = \epsilon\vec{E}$

magnetic susceptability in a plasma $\mu = \mu_0 = 4\pi\times10^{-7}$

vector potential

and $\vec{E} = -\nabla\phi - \frac{\partial\vec{A}}{\partial t}$ or $\phi = \int\vec{E}\cdot d\vec{l} + \frac{\partial}{\partial t}\int\vec{B}\cdot\hat{n}\,da$

electric potential

and $\vec{B} = \nabla\times\vec{A}$

This is a review of all of Electricity and Magnetism in 3 pages (gulp)

If you combine 4–7 you get in vacuum the wave equation for light

Now lets get back to what makes a plasma a plasma.

Ans. if there are many-particles they can be have collectively its the additional terms in equations 2 and 3 that do it.

How many particles are needed.

We will derive this but first let us give you the answer It is not only how many particles it is how many /unit volume If you spread 10^{10} particles in the solar system you wont have a plasma. If you put them in our chamber you will!

Debye length $\lambda_D = \left(\frac{ne^2}{\epsilon_0 kT}\right)^{1/2} = 740\sqrt{\frac{T}{n}}$ cm T = temp in e.V. $n = \#/cm^3$

k = Boltzman's constant $= 1.38\times10^{-23}$ Joule/°K

$\epsilon_0 = 8.85\times10^{-12}$ F/m (F = Farad)

e = electron charge $= 1.6\times10^{-19}$ Coulomb

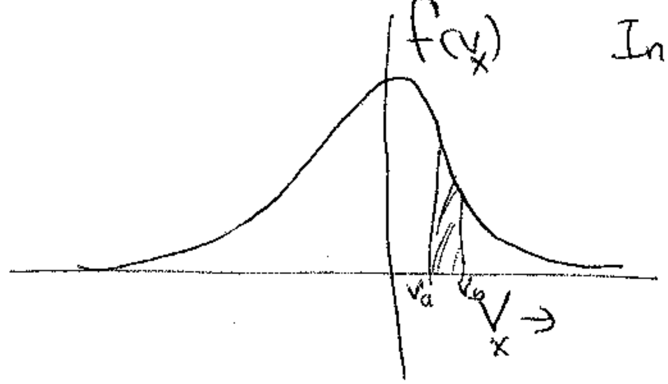
For laptay plasma assume 2eV $\lambda_D = 740\left(\frac{2}{10^{10}}\right)^{1/2} = 1.05\times10^{-2}$ cm $\approx .10$ mm

If we talk about things happeing at $x < \lambda_D$ then we have a bunch of charged particles. For $x > \lambda_D$ the plasma acts collectively

	n	T(eV)	λ_D
Interstellar gas	1	1	700 cm
Solar Corona	$10^9/cm^3$	100	0.2 cm
Thermonuclear plasma	10^{15}	10^4	.002cm
Laser plasma	10^{20}	100	7×10^{-7} cm

Now where does λ_D come from?

In order to find how the Debye length comes about we must first foray into statistics. The plasma is made of many charges all moving about randomly. At any instant each particle has velocity $\tilde{v}$



In 1 Dimension

The area under the shaded patch is $\int_{V_a}^{V_b} f(v_x)\,dv_x$

is the number of particles moving in the +x direction with velocitys between V_a and V_b

Therefore $\int_{-\infty}^{+\infty} f(v_x)\,dv_x = n(x,t)$

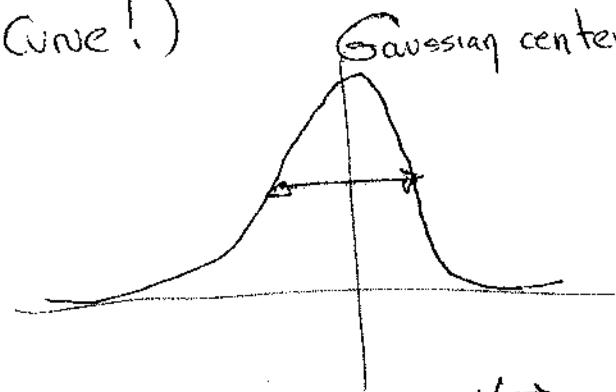
↑ total number of particles at position x and time t

This means that in full three dimensions

$f = f(\tilde{r}, \tilde{v}, t)$ a function of $\underline{\underline{7}}$ variables

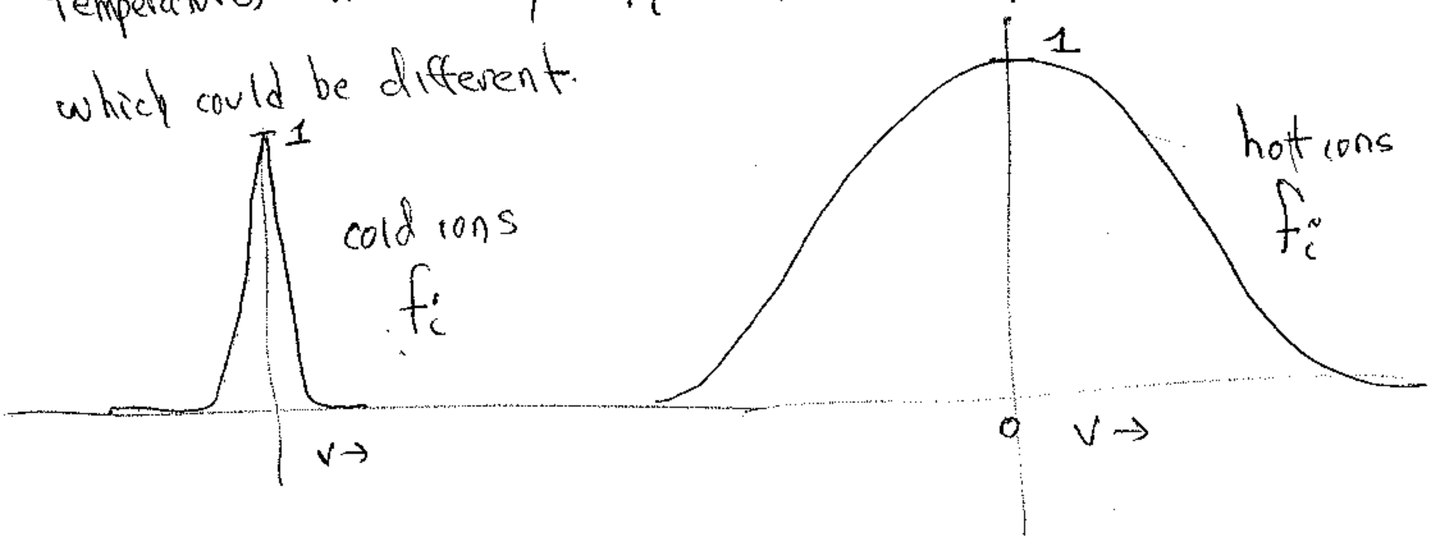
f is really a probability of finding so many particles at a particular position $\tilde{r}$, and velocity $\tilde{v}$ at time t.

We state without proof here that if all the particles are in thermal equilibrium then the shape of the distribution function f_i is a Gaussian (like the old IQ Bell Curve!)



The full width at half maximum is related to the spread in velocities or the temperature

Also since the electrons and ions may have different temperatures there is an $f_i(\vec{r},\vec{v},t)$ and $f_e(\vec{r},\vec{v},t)$ which could be different.



If we say the plasma is in thermal equilibrium then:

$$f(\vec{r},\vec{v},t) = f(r,t)\, A e^{-\frac{1}{2}mv^2/kT} \quad (8)$$

$\leftarrow$ temperature

Boltzman's constant

Gaussian shape

The integral of f over all space and velocities must (7)
be the density at a fixed time

*

(9) $$\int \underbrace{f(\vec{r},\vec{v},t)\, d^3v\, d^3r}_{dv_x dv_y dv_z\, dx dy dz} = n(t)$$

in rectangular coordinate

If the plasma density does not change in time $n = \underbrace{n_0}_{\frac{\text{number}}{\text{volume}}}$

In the special case when one of the species (lets say the ions) is much colder than the other we can assume they are not moving and the density is constant everywhere and it is constant.

(To prove it from 8 you need to use the dirac delta function!)

Since a plasma is electrically neutral on the average

$n_e = n_i$ It is very small deviations from this that are responsible for a great many phenomena

* note A is defined such that $\displaystyle\int_{-\infty}^{+\infty} f(v)\, d^3v = \int_{-\infty}^{-\infty} A e^{-\frac{1}{2}mv^2/kT}\, d^3v = 1$

(8) One last note. Since negative charges repel each other, etc we will have more or less particles in a spot or another depending upon the local potential. So for charged particles we must rewrite 8 as

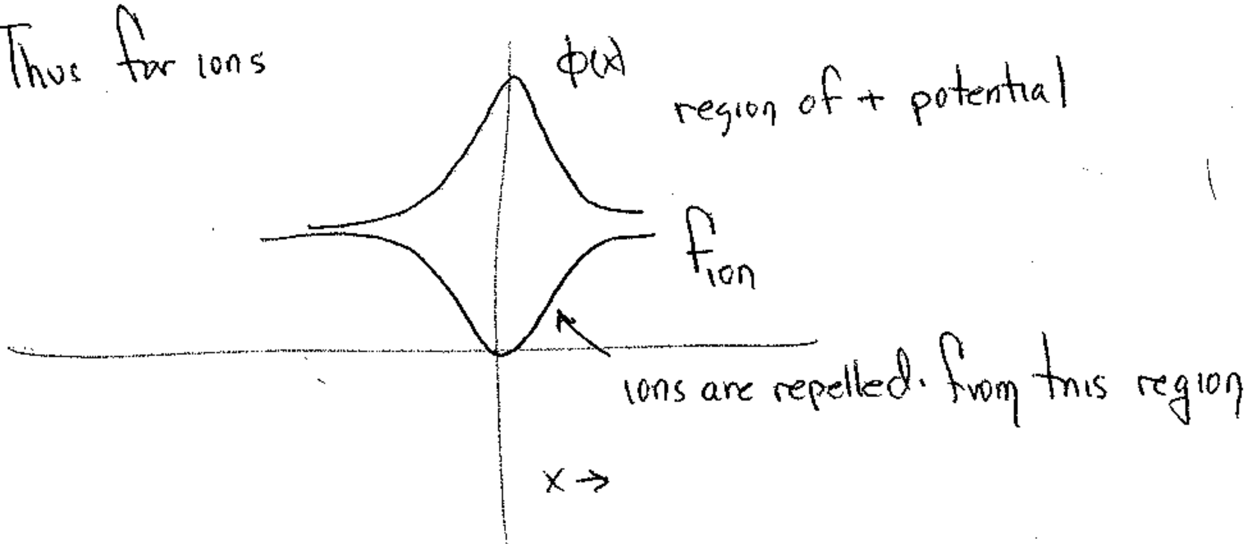
$$f = n_0 \left(\frac{m}{2\pi kT}\right)^{3/2} e^{-\frac{1}{2}mv^2/kT_e} \; e^{-q\phi/kT}$$

ϕ = electric potential $\quad \phi = \phi(x)$ or $\phi = \phi(\vec{r})$

$q = +e$ ions of charge 1

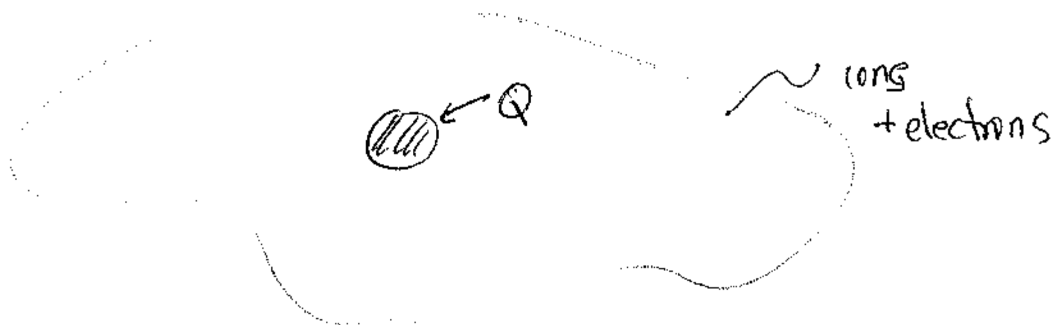
$q = -e$ electrons

Thus for ions



Now we can attack the debye problem

Suppose we have a neutral plasma and we stick a ball of charge Q into the middle of it



what happens.

We must solve for the electric field

$$\epsilon_0 \nabla \cdot \vec{E} = \rho = q_{ion} n_i + q_{elec} n_e = e[n_i - n_e]$$

ρ: charge density

But $\vec{E} = -\nabla \phi$ and thus

ϕ: potential

$$\epsilon_0 \nabla \cdot \vec{E} = \nabla \cdot (-\nabla \phi) = -\nabla^2 \phi$$

Thus

$$\epsilon_0 \nabla^2 \phi = -e[n_i - n_e]$$

This is called Poisson's equation

In rectangular coordinate

$$\nabla^2 \phi = \frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} + \frac{\partial^2 \phi}{\partial z^2}$$

But since we are sticking a <u>ball</u> of charge into a plasma lets use spherical coordinates
If we assume that disturbance will be a function of r only (no θ or ϕ dependence since the +Q charge is a ball)

$$\epsilon_0 \nabla^2 \phi = \frac{\epsilon_0}{r^2}\frac{\partial}{\partial r} r^2 \frac{\partial}{\partial r}\phi = -e[n_i - n_e]$$

Now the NEXT assumption. Suppose the ions are ice cold and form a uniform background and the electrons can move (Remember $m_e << m_I$ so they are faster!)

Then $n_i = n_0$

and $n_e = n_0 e^{-q\phi/kT_e} = n_0 e^{+e\phi/kT_e}$ — electron temperature ($q = -e$)

Now our equation is

$$\boxed{\epsilon_0 \frac{1}{r^2}\frac{\partial}{\partial r} r^2 \frac{\partial}{\partial r}\phi(r) = + n_0 e\left[e^{e\phi/kT_e} - 1\right]}$$

This is highly non linear why?