Non-local Ohm’s law during collisions of magnetic flux ropes

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(Received 20 March 2017; accepted 6 June 2017; published online 27 June 2017)

Two kink unstable magnetic flux ropes are produced in a carefully diagnosed laboratory experiment. Using probes, the time varying magnetic field, plasma potential, plasma flow, temperature, and density were measured at over 42,000 spatial locations. These were used to derive all the terms in Ohm’s law to calculate the plasma resistivity. The resistivity calculated by this method was negative in some spatial regions and times. Ohm’s law was shown to be non-local. Instead, the Kubo resistivity at the flux rope kink frequency was calculated using the fluctuation dissipation theorem. The resistivity parallel to the magnetic field was as large as 40 times the classical value and peaked where magnetic field line reconnection occurred as well as in the regions of large flux rope current.

Published by AIP Publishing. [http://dx.doi.org/10.1063/1.4990054]

Flux ropes are magnetic structures with helical magnetic fields and currents and are found in astrophysical plasmas throughout space. Flux ropes routinely occur within the Earth’s magnetopause boundary layer and also within the magnetotail. The surface of the sun is littered with arched flux ropes, which are visible as arched filaments when viewed in ultra violet or X ray emissions. It is common in nature for flux ropes to have one or more companions. When two or more flux ropes are close to one another, they can interact often in complex ways and collide. During a collision, a small component of the magnetic field from each rope can be anti-parallel, and magnetic field line reconnection can be triggered. When reconnection is two-dimensional, a magnetic null can exist with field lines piling into it and reconnecting. This occurs in the magnetotail and has been observed in numerous laboratory experiments. Generally, in nature, the magnetic fields are three-dimensional and there are no nulls as in the 2D case, but reconnection happens nevertheless. Arched magnetic flux ropes with helical fields have been studied in the laboratory, but here, we consider a simpler geometry, with two side-by-side flux ropes generated from parallel currents in a background magnetic field. The ropes are kink unstable and periodically smash into one another, and each time they do reconnection occurs. Ohm’s law links the local plasma resistivity to the total electric field and current. In this work, we measure all the quantities necessary to calculate every term in Ohm’s law and examine the resulting resistivity.

These experiments were performed in the Large Plasma Device (LaPD) at UCLA. The plasma is produced by a DC discharge between a Barium Oxide coated cathode and a mesh anode. The bulk of the plasma carries no net current and is therefore quiescent. The background plasma parameters are as follows:

\[
\frac{\Delta n}{n} \approx 3\%, \ 0.5 \leq T_e \leq 5 \text{ eV}, \ 10^{10} \leq n_e \leq 2 \times 10^{12} \text{ cm}^{-3}, \ T_i \approx 1 \text{ eV}.
\]

The parallel Lundquist number is \(\sim 1.8 \times 10^5\), which is based on Spitzer resistivity and an 11 m rope length. The background plasma is pulsed at 1 Hz with a typical plasma duration of 15 ms. A schematic of the machine indicating how the background plasma and flux ropes are generated is shown in Fig. 1.

The background plasma is generated by an oxide coated cathode schematically shown on the left of Fig. 1. A transistor switch in series with a capacitor bank is used to pulse a voltage between the negatively biased cathode and an anode located 50 cm away. The background plasma is 18 m in length and 60 cm in diameter. A second, high emissivity cathode (LaB6) is inserted in the far side of the machine. The flux rope cathode is masked with a carbon sheet into which two circular holes are cut 3.5 cm in radius and 1 cm apart edge-to edge so it can only emit from the unmasked areas. The cathode mask is 32 cm from the second cathode. A second transistor switch, capacitor bank, and charging supply are used for the ropes. The reference probe was used to calculate the temporal autocorrelation time for the magnetic field. We can reliably reconstruct the data for 4 ms, about twice the time in our datasets.

The plasma in the ropes is highly ionized with a temperature of 12 eV. The electrical systems, which generate the ropes, and background plasma are independent of one another. Diagnostics include Langmuir probes, Mach probes (flow), and three-axis magnetic pickup probes. A specialized emissive probe measured the plasma potential, \(V_p(t)\), at each of the 42,150 spatial locations. The time series was digitized at 16,000 timesteps. Subsequently, \(V_p\) was corrected for the electron temperature, which was also measured at each location. The gradient of the plasma potential was used to determine the space charge contribution to the electric field.

Ohm’s law has long been used to estimate the plasma resistivity in the analysis of spacecraft data, in the interpretation of laboratory experiments and in computer simulations. Often, one or more terms are dropped using symmetry arguments or assumptions on the relative size of terms. The justification for this is often questionable.
Ohm’s law for electrons is in SI units

\[
\frac{m_e}{ne^2} \frac{d\vec{J}}{dt} = \vec{E} + \vec{u} \times \vec{B} + \frac{1}{ne} \vec{\nabla} \cdot \vec{P} - \frac{1}{ne} \vec{J} \times \vec{B} - n \eta |\vec{J}| - \eta_\perp \vec{J}_\perp.
\]

(1)

Here, \(\vec{P}\) is the pressure tensor, \(n\) is the plasma density, \(\vec{J}\) is the current (in this experiment, it is carried by the electrons), \(e\) is the absolute value of the electron charge, and \(\vec{B}\) is the total magnetic field. The resistivity, \(\eta\), is different along and across the magnetic field. In the steady state and when there is no magnetic field or pressure, Ohm’s law reduces to the familiar \(\vec{E} = \eta \vec{J}\). For the first time to our knowledge, all the terms in the generalized Ohm’s law, Eq. (1), to determine the resistivity, were measured as a function of space and time in a flux rope experiment. The goal was to derive the parallel resistivity from the data. When the dot product of Eq. (1) with the local magnetic field (the background field and that of the ropes) is taken, one arrives at

\[
\vec{E} \cdot \vec{b} + \frac{1}{ne} \vec{\nabla} P \cdot \vec{b} = \eta |\vec{J}|\parallel \quad \text{with} \quad \vec{b} = \frac{\vec{B}}{|\vec{B}|}.
\]

(2)

In Eq. (2), \(\vec{b}\) is a unit vector along the local value of the magnetic field due to the external magnets and plasma current and \(P\) is the scalar pressure. The time varying current term in Eq. (1) is negligible for our experimental parameters.

The perpendicular magnetic field can be as large as 30 G or about 1/10 of the background field, \(B_{B0} = 330\) G. The flux ropes rotate about each other in addition to being linearly kink unstable. As the rope rotation starts at a slightly different time for each experimental shot, a conditional averaging technique was used to define the experimental start time, which differs from the time the rope currents are switched on.

The resulting magnetic field lines write about themselves and twist about one another as previously observed. When the kink motion drives the ropes into one another, there are bursts of reconnection and a Quasi Separatrix Layer (QSL) forms. QSLs are regions in which the magnetic field connectivity changes rapidly but continuously across a narrow spatial region. Reconnection results in an induced electric field which, in principle, can drive reverse currents in the plasma. The three-dimensional plasma currents are evaluated from the magnetic field using \(\nabla \times \vec{B} = \mu_0 \vec{J}\).

The plasma density was measured everywhere in the volume by collecting ion saturation current to the six faces of a Mach probe. The ion saturation current, \(I_{sat}\), to a probe is proportional to \(I_{sat} \propto n \sqrt{T_e}\). The electron temperature was determined by sweeping the Langmuir probe I-V characteristics at thousands of spatial locations. The contours of electron temperature calculated closely matched contours of constant current on these planes and at the times the probe was swept. The background temperature (where there was no current) was 4 eV. Figure 2 shows the scalar plasma pressure, \(P = n(\vec{r}, t)K T_e(\vec{r}, t)\) at one time \(t = 5.5865\) ms. The ropes are switched on at \(t = 0\), and in the first few meters, the iso-surface of constant pressure \((P = 5 J/m^3)\) mirrors that of the rope magnetic field. Isosurfaces of lower pressure fill more of the volume. The proper term in Ohm’s law is the divergence of the pressure tensor. In the experiment, the electron-electron collisional relaxation rate is 5 MHz or about one thousand times faster than time scales associated with the flux rope motion. These collisions isotroprise the pressure tensor. A typical mean free path for collisions between 10 eV electrons is 21 cm, far smaller than the length of the ropes. Finally, the large guide field, in comparison to the transverse flux rope fields, suppresses meandering orbits of electrons in the transverse plane, which lead to tensor pressures. Consequently, we estimate that the pressure is well approximated by its measured scalar value.

The electric field has two components, \(\vec{E} = -\nabla V_p - \frac{\partial \vec{A}}{\partial t}\).

The space charge component is often neglected. This is not the case here. In Fig. 2(b), the axial electrostatic field is negative, in the same direction as \(J_0\) inside the current channels and is of order ten times larger than the inductively induced electric field [represented by the second term, \(-\frac{\partial \vec{A}}{\partial t}\) and shown in Fig. 2(c)]. It is also evident that the two components have different spatial morphologies. The induced electric field has the opposite sign as the electrostatic field in the region where the currents are large, and this is expected when reconnection occurs.
The parallel and transverse resistivities were evaluated plane by plane, and they will be discussed in detail in a much longer paper in the future. Estimating the error in each measurement of the terms in Eq. (2) and the total error produced by adding and dividing them, we estimate that the experimental error in the measurement of $\eta_{jj}$ is 20%. Here, we present in Fig. 3 the parallel resistivity at a fixed time on two transverse planes.

Contours of field aligned $J_{||}$ are also shown in Fig. 3. The temporal appearance of negative resistivity has nothing to do with reconnection and is not observed in the QSL region. The field-aligned resistivity was also calculated and also has negative regions. The volume-averaged resistivity is always positive. Reverse currents have been seen in the past in experiments on reconnection. There are axial pressure gradients that can potentially drive reverse currents, but these are measured and were included in the Ohm’s law evaluation. Assuming that there is no dynamo, one may question whether Ohm’s law can be used at all in these circumstances. It is possible that the contributions to Ohm’s law are non-local and Eq. (1) cannot be used. Non-locality occurs when fields and pressure gradients along a moving field line all contribute to the resistivity at a remote point. Ohm’s law requires a point-by-point balance between momentum gain from the electric field and loss due to scattering. Jacobson and Moses discovered that this loss and gain may be in global balance but not locally. The criterion for this is governed by a parameter $\alpha$

$$\alpha = 2L \sqrt{\frac{\partial B_{||}}{B}} \left[ \frac{1}{E_{||}} \nabla \cdot E_{||} \right]^2.$$  

Here, $E_{||}$ is the local electric field parallel to a magnetic field line and $L$ is the parallel decorrelation length along the field.

The first term, $2L \sqrt{\frac{\partial B_{||}}{B}}$, is the mean-square cross field excursion for an electron which scatters 90° traversing $L$. The second is the inverse of the characteristic length for variation of $E_{||}$. Here, we took the smallest length possible, the electron mean free path in the center of the flux rope. When $\alpha \ll 1$, the local version of Ohm’s law is correct. This term is evaluated and shown in Fig. 4. As the quantities in Eq. (3) vary in time, alpha was averaged over one rope oscillation (from 5.104 ms < $t$ < 5.321 ms). Alpha is as high as 100, the average over the total plasma volume $\langle \alpha \rangle = 6$.

Figure 4 indicates that the non-locality condition is satisfied and Ohm’s law as expressed in Eq. (1) cannot be used to evaluate the parallel resistivity.

Since it was determined that Ohm’s law cannot be used, we adopt another method to determine the global resistivity,
which is based on the fluctuation-dissipation theorem.\textsuperscript{28} The AC plasma conductivity tensor was derived by Kubo\textsuperscript{29} and is given by

$$\sigma(\omega, \vec{r}) = \frac{n e^2}{m v_{th}^2} \int e^{-i \omega \tau} v_\nu(t, \vec{r}_0) v_\mu(t + \tau, \vec{r}) d\tau dt$$  \hspace{1cm} (4)

and we apply this formula under the assumption of non-equilibrium steady state in which the currents lead to dissipation.\textsuperscript{30,31} Here, $n$ is the plasma density, $v_\nu$ is a component of the velocity ($\nu = x, y, \text{ and } z$), and $v_{th}$ is the local electron thermal velocity of the bulk plasma at the movable probe location $\vec{r}$. Equation (4) is a double integral first over the delay time $\tau$ and then $t$ for the Fourier transform. This is a correlation function in which the index $\nu$ pertains to a velocity component at a fixed point, $\vec{r}_0$, chosen to lie on a field line in the center of the average current of one of the ropes. The other index $\mu$ is that of the velocity field at all locations, $\vec{r}$, in the x-y plane for a given z location. The velocity in the integral is the electron drift, which can be obtained from the current density $\vec{J} = ne\vec{v}$, which is also measured throughout the volume. We take this to be the “test particle” velocity in the Kubo formula. This is an approximation and we have assumed that the current is from a drifting Maxwellian. The tensor conductivity, both the real and imaginary parts, was evaluated at 5 kHz, the rope oscillation frequency. Note that all relevant quantities in this experiment, such as $Q$, magnetic helicity, and flux rope positions, oscillate at this frequency as well.

The Kubo resistivity, $1/\sigma_{zz}$ (the axis parallel to $B_0$ and the device axis), on four planes is shown in Fig. 5. In Figs. 5(a) and 5(b), the two current channels are positioned at the upper left and right hand corner. The resistivity is also large in regions where the current is small, for example, the edges of the plane in Fig. 3. Let us focus on the central region where current is appreciable and reconnection occurs. In the first 4–5 m of the interaction region, the collisions of the ropes are most prominent and it is most likely that the bulk of reconnection occurs there. This is reflected in the larger resistivity in the center of Figs. 5(a) and 5(b). There are also large resistivities inside the current channels and in Fig. 5(c), $\eta$ peaks inside the currents channels, not in the gradient region. In Fig. 5(d), at $\delta z = 8.32$ m, the currents have nearly merged. The enhanced resistivity in the reconnection region and in the center of the current channels can arise from different processes.

The measurement of the plasma resistivity in a current carrying plasma is difficult. The worst thing one can do is to
divide the external voltage imposed on the system by the current through it. It is a simple matter to measure the current but the external voltage has no relation to the internal electric field due to sheaths, which always develop on the boundaries. Experiments have tried to correct for the boundary effects, but one cannot be sure that the corrections are valid. Some experiments have used Ohm’s law with assumptions in order to neglect terms to estimate the resistivity. It is not clear if their conclusions are valid. The closest experiment to this one was reported by Intrator et al. The flux ropes were produced by two plasma guns at the end of a vacuum vessel with an axial field and there was no background plasma. The measurement relevant quantities were spatially sparse making integration along field lines impossible. The value of the global resistivity was estimated from an incomplete Ohm’s law and was of order 1.3 larger than classical. The Kubo resistivity has been used in computer simulations of a 3D magnetic null point to calculate the plasma resistivity. The simulation included chaotic orbits at the null point where there was no exact method to calculate the resistivity.

In the experiment reported here, all the terms in Ohm’s law have been evaluated in space and time using the most comprehensive data set on magnetic flux ropes acquired to date. Ohm’s law yielded a non-physical negative resistivity at some spatial locations. This was also the case when the resistivity was evaluated by integrating the electric field and plasma current along field lines. We examined the possibility that Ohm’s law was non-local and could not be used to evaluate \( \eta \). This turned out to be the case where in the future those who use it in experimental or simulation results should do so very carefully.

The AC Kubo conductivity was derived from the data and the parallel resistivity evaluated. The AC resistivity is enhanced in the flux rope currents and in the region between the current channels at a location where the QSL is large and reconnection occurs. What is the cause of anomalous resistivity (that is anything larger than the classical one)? The culprit is generally thought to be scattering of the electrons by waves or turbulence. There are a number of candidates such as ion acoustic turbulence and Langmuir waves, basically any phenomenon that has a large fluctuating electric field. In this experiment, the quantities measured to calculate the terms in Ohm’s law were filtered to exclude everything above 3 MHz. With high frequency probes \((2.5 \text{MHz} \leq f \leq 1 \text{GHz})\), we measured the spectra of electric and magnetic field fluctuations. The electric field spectra are exponential from 1 to 10 MHz and have a broad peak at 20 MHz which is \(-20 \text{dB}\) from that of the rope oscillations. A second broad peak at 80 MHz (near the lower hybrid frequency) is 15 dB below the first. Based on this, we think that there are little, if any, high frequency contributions to the resistivity.

One of the authors (W.G.) would like to thank George Morales for interesting discussions. We also would like to thank Zoltan Lucky, Marvin Drandell, and Tai Ly for their expert technical support. The work was funded in part by a Grant from the University of California, Office of the President (12-LR-237124). It was performed at the Basic Plasma Science Facility, which is funded by DOE (DE-FC02-07ER54918) and the National Science Foundation (NSF-PHY 1036140).


