On generation of Alfvénic-like fluctuations by drift wave–zonal flow system in large plasma device experiments

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According to recent experiments, magnetically confined fusion plasmas with “drift wave–zonal flow turbulence” (DW-ZF) give rise to broadband electromagnetic waves. Sharapov et al. [Europhysics Conference Abstracts, 35th EPS Conference on Plasma Physics, Hersonissos, 2008, edited by P. Laloussis and S. Moustazis (European Physical Society, Switzerland, 2008), Vol. 32D, p. 4.071] reported an abrupt change in the magnetic turbulence during L-H transitions in Joint European Torus [P. H. Rebut and B. E. Keen, Fusion Technol. 11, 13 (1987)] plasmas. A broad spectrum of Alfvénic-like (electromagnetic) fluctuations appears from $\mathbf{E} \times \mathbf{B}$ flow driven turbulence in experiments on the large plasma device (LAPD) [W. Gekelman et al., Rev. Sci. Instrum. 62, 2875 (1991)] facility at UCLA. Evidence of the existence of magnetic fluctuations in the shear flow region is shown. We present one possible theoretical explanation of the generation of electromagnetic fluctuations in DW-ZF systems for an example of LAPD experiments. The method used is based on generalizing results on shear flow phenomena from the hydrodynamics community. In the 1990s, it was realized that fluctuation modes of spectrally stable nonuniform (sheared) flows are non-normal. That is, the linear operators of the flows modal analysis are non-normal and the corresponding eigenmodes are not orthogonal. The non-normality results in linear transient growth with bursts of the perturbations and the mode coupling, which causes the generation of electromagnetic waves from the drift wave–shear flow system. We consider shear flow that mimics tokamak zonal flow. We show that the transient growth substantially exceeds the growth of the classical dissipative trapped-particle instability of the system. © 2009 American Institute of Physics. [doi:10.1063/1.3211197]

I. INTRODUCTION

In magnetically confined fusion plasmas, the reduction in cross-flow transport by shear flow is now widely utilized. A variety of measurements have confirmed the role of flow shear in impeding the transport and diminishing the intensity of turbulence of fusion plasmas in all geometries and regimes.1–5 Moreover, record values of confinement time and the ratio of fusion power output to input heating power have been achieved using flow-shear-induced transport barriers in tokamak/Joint European Torus (JET) plasmas.6 Consequently, the importance of sheared $\mathbf{E} \times \mathbf{B}$ flows to the development of the L-H transition and internal transport barriers is now widely appreciated. As a result, for more than a decade, the study of shear-flow-induced phenomena has been central in fusion research.

It is now accepted that zonal flow (ZF) is a crucial player in the mechanism for self-regulation of drift wave (DW) turbulence and transport in nearly all cases and regimes of fusion plasmas, so this type of turbulence is now commonly referred to as “drift wave–zonal flow (DW-ZF) turbulence.” Both theoretical work and numerical simulation made important contributions to this paradigm shift and to the study of the dynamics of DW-ZF turbulence systems. Specifically, numerical simulations confirm the experimental observation that DW turbulence and transport levels are reduced when the ZF is properly generated. In contrast, the fluctuations themselves lack such universality, changing character from device to device and within any given device in different regimes.7–11 Specifically, the recent JET (Ref. 7) and large plasma device (LAPD) (see below Sec. III) experiments indicate that DW-ZF turbulence gives rise to a broadband spectrum of electromagnetic waves, which is the subject of the present study. ZF is different from externally imposed shear $\mathbf{E} \times \mathbf{B}$ flow in that ZF is a self-organized turbulence-driven phenomena and it is bipolar. However, since locally shear $\mathbf{E} \times \mathbf{B}$ flow acts much like tokamak ZF, the two will be treated interchangeably.

Sharapov et al.7 reported abrupt changes in magnetic turbulence during L-H transitions in JET plasmas. The fluctuation data show a broad spectrum of electromagnetic waves in the presence of large ZF. Alfvénic-like fluctuations appear from $\mathbf{E} \times \mathbf{B}$ flow driven turbulence in experiments on the LAPD facility at UCLA. The LAPD (see Fig. 1 of Refs. 10–12) is 18 m in length, with a singly ionized helium plasma column approximately 0.8 m in diameter created by a pulsed discharge 20 ms long from a barium oxide coated emissive cathode. A radial electric field is established by bi-
using the chamber wall with respect to the anode and cathode for approximately 5 ms during the discharge. The radial electric field profiles can be varied by changing the bias voltage and the magnetic field boundary conditions within the LAPD. The experiment is described further in Sec. III and Ref. 11. A strong radial electric field, in turn, induces an $\mathbf{E} \times \mathbf{B}$ plasma rotation in this region of the plasma. This wall-to-cathode biasing results in different sheared $\mathbf{E} \times \mathbf{B}$ flows and, consequently, different turbulent regimes at the edge of the plasma column. For some parameters of the LAPD, it appears that the DW-ZF system is enriched by high frequency Alfvénic-like fluctuations.10,11 Generation of broadband electromagnetic fluctuations in DW-ZF systems is a significant phenomenon because electromagnetic fluctuations can modify the anomalous transport. In addition, the characteristics of these fluctuations indirectly give information about the dynamics of the DW-ZF system.

In the LAPD experiments, the ZF is generated by external mechanisms and may be stronger than ZF generated by self-regulation mechanisms, such as in tokamaks. Specifically, ZF amplitude is reported as $U_E \approx 4 \times 10^5$ cm/s, the radial size of the ZF region is $L_E \approx 5$ cm, and the Alfvén speed is $V_A \approx 9 \times 10^7$ cm/s. The reported Alfvénic-like fluctuations occur at high shear rates of the ZF velocity (see Sec. III). It will be shown that fluctuations in flows with high shear rates are strongly non-normal. The strong non-normality results in linear transient growth with bursts of the perturbations and mode coupling, which is how electromagnetic waves are generated from a DW-ZF system. For the above values, the flow-shear parameter is $A = U_E/L_E \approx 0.8 \times 10^5$ s$^{-1}$. Considering perturbations with characteristic size along the axial magnetic field of the order of the length of the device $L_i \approx 1/k_i \approx 10^5$ cm, the normalized shear parameter is of the order of unity,

$$S = \frac{A}{k_i V_A} \approx 0.9.$$ (1)

The aim of this paper is to describe a possible mechanism for the generation of electromagnetic waves in DW-ZF systems on an example of LAPD experiments. We will show that this phenomenon is universal at high shear rates of ZF and should also take place in tokamaks. In tokamaks, the $S$ parameter is peaked on the rational surfaces. This study extends the hydrodynamic stability theory of nonuniform flows, which saw a great deal of development in the 1990s, to DWs. The regime, where the dimensionless shear parameter $S = A/k_i V_A \approx 1$, sees strong shear modification of the DW turbulence. The non-normality of sheared flow and the inadequacy of the modal/spectral approach are addressed by some plasma physicists. There is an interesting study by Cooper,11 for instance, that notes shear flow non-normality and numerically describes the transient character of linear time evolution of azimuthal perturbations for a case of sheared toroidal rotation where the flow is parallel to the background magnetic field. In contrast, the present study addresses the case where the shear flow is perpendicular to the background magnetic field. By solving for the Floquet modes of the fluctuations, Cooper showed an example of bursts of wave energy that are periodic in time, which resemble fishbone-type instabilities in tokamaks. However, by examining the review articles,1,5 it seems that the breakthrough of the 1990s of the hydrodynamic community had limited impact on the plasma and magnetic fusion communities.

To demonstrate the relevance of this novel approach of the hydrodynamic community to the dynamics of fusion zonal/shear flows and to join a bibliography of the subject, some salient results are presented in Sec. II. In Sec. III the essence of the LAPD experiments and some important graphs of experimental results are presented. Section IV gives dynamical equations describing local linear dynamics of perturbations sheared $\mathbf{E} \times \mathbf{B}$ flow of LAPD experiments, introduces the physical approximations and the mathematical formalism, and describes characteristics of perturbations in the absence of shear flow. In Sec. V we present the qualitative and quantitative analyses of the linear dynamics of perturbations at small and high shear rates of ZF. We show how the shear flow generates the Alfvénic fluctuations through (the shear flow-induced) coupling to DWs. We summarize and discuss the results in Sec. VI. The set of model equations that describe the linear dynamics of perturbations on LAPD experiments is derived in Appendix A. The initial conditions for numerical simulations of the basic equations are derived in Appendix B.

II. SHEAR FLOW “NON-NORMALITY” AND ITS CONSEQUENCES

Investigating shear flow dynamics implies the following steps: introduction of perturbations into a mean flow, linearization of the governing equations, and description of the dynamics of perturbations and flow using the solutions of the initial value problem (i.e., following temporal balances in the flow). This can be done, in principle, but in practice it is a formidable task. Therefore, a common simplification in many stability calculations is the assumption of exponential time dependence, also referred to as the normal-mode (or modal) approach. This ansatz transforms the linear initial value problem into a corresponding eigenvalue problem. The imaginary part of the computed eigenvalues represents an exponential growth rate. The basic flow is labeled unstable if an eigenvalue is found in the upper half of the complex plane. In time, this approach became the canonical methodology and resulted in a focus on the asymptotic stability of the flow with a lack of attention to initial values or to the finite time period dynamics. The finite time period dynamics were considered insignificant and usually left to speculation.

According to classical fluid mechanics,14 the existence of an inflection point in the equilibrium velocity profile is a necessary condition for the occurrence of a linear (spectral) instability, corresponding to exponentially growing solutions, in hydrodynamic flows. Thus smooth shear flows (flows without an inflection point) are spectrally stable. However, it is well known from laboratory experiments and from numerical simulations that perturbations may cause a transition from laminar to turbulent state in spectrally stable flows (e.g., Couette flow). Specifically, difficulties in the modal analysis of linear processes in shear flows were rigorously
revealed in the 1990s. It has been shown that the operators from the modal analysis of linear processes in shear flows are exponentially far from normal. (The norm of the flow non-normality increases with the shear rate.) Consequently, the eigenfunctions are not orthogonal to each other and strongly interfere. As a result, even when all the eigenfunctions decrease monotonically in time (i.e., all eigenfrequencies are in the lower half of the complex plane), a particular solution may indicate a large relative growth in a limited time interval. That is, a superposition of decaying normal modes may grow initially, but will eventually decay as time goes by. Thus, analysis of eigenmodes and eigenfunctions as linearly independent is misleading. The physics of the shear flow non-normality induced transient growth of vortical perturbations is presented in Ref. 16 (by means of conservation laws—the “lift-up” mechanism) and Ref. 17 (by means of dynamical equations on an example of fluid parcel/particle dynamics). The potential for this transient growth has been recognized for more than a century (see Refs. 18–20). However, the importance of this phenomenon was only understood in the 1990s. It was shown that transient growth can be significant for spectrally stable flows and that its interplay with nonlinear processes can result in the transition to turbulence.

The modal approach dominated the study of disturbances for many decades. Only until the 1990s were its limitations recognized and its shortcomings linked to the above-mentioned behavior. As a result, novel ways of describing fluid stability emerged (see Refs. 21–24), which allow the quantitative study of short-term disturbance behavior. These approaches have enjoyed substantial success in furthering a more complete understanding of instabilities and in providing the missing linear and nonlinear dynamics in a variety of shear flows. As an alternative to the modal approach, the system is treated as an initial value problem. Two different formulations of the nonmodal approach will be described. The first formulation of the nonmodal approach, the Kelvin mode approach (stemming out of the 1887 paper by Lord Kelvin) has become well established and has been extensively used since the 1990s. The Kelvin modes represent the “simplest element” of shear flow physics, and have also been referred to as “flowing eigenfunctions” in expanding fluctuations. Strictly speaking, the Kelvin mode approach describes systems with constant shear flow, but, nonetheless, it also guides the understanding and qualitative description of smooth shear flow phenomena. In particular, the Kelvin approach correctly describes transient exchange of energy between basic shear flow and perturbations. It exposes two novel channels of linear coupling of perturbation modes in shear flows.

(a) The first energy transfer channel is resonant by nature and leads to energy exchange between different wave modes. The mutual transformation of different kinds of waves is studied numerically and analytically in detail for magnetohydrodynamic waves. The mutual transformation occurs at small shear rates if the dispersion curves of the wave branches have pieces nearby one another.

(b) The second energy transfer channel is nonresonant (vortex and wave mode characteristic times are significantly different) and nonsymmetric (a vortex mode is able to generate a wave mode but not vice versa). This channel leads to energy exchange between vortex and wave modes, as well as between different wave modes. Chagelishvili discussed flow non-normality induced phenomena, including the nonresonant coupling mechanism, in detail and presented several simpler examples. We concentrate on this channel of mode coupling because it is important at high shear rates, like the shear rates observed in the LAPD experiment [see Eq. (1) in Sec. I]. In the present study, the Kelvin mode approach will be used.

The second formulation of the nonmodal approach is a generalized stability theory (GST) that extends modal stability theory to comprehensively account for all transient growth processes in nonuniform flows by including the interaction among modes and the mean flow regardless of whether the modes are unstable or not. GST introduces a finite time horizon over which an instability is observed. Finite-time stability analysis markedly deviates from the traditional Lyapunov stability concept, and it is not surprising that the disturbance that grows the most over a short time scale differs significantly from the least stable mode. Given a finite time horizon, GST can be employed to quantitatively describe the perturbation dynamics. GST reveals a remarkably rich picture of linear disturbance behavior that differs greatly from the solutions of the modal approach. GST is easily extendable to incorporate time-dependent flows, spatially varying configurations, stochastic influences, nonlinear effects, and flows in complex geometries.

One should also mention that during the 1990s, a new viewpoint emerged in the hydrodynamics community on understanding the onset of turbulence in spectrally stable shear flows, labeled as “bypass” transition (cf. Refs. 27, 30, and 42–52). Although the bypass transition scenario involves nonlinear interactions, which intervene once the perturbations have reached finite amplitude, the dominant mechanism leading to these large amplitudes appears to be linear. This bypass transition concept is based on the linear transient growth of vortical perturbations. The bypass concept implies that the energy extracted from the basic flow by linear transient mechanisms causes the increase in the total perturbation energy during the transition process. The nonlinear terms are conservative and only redistribute the energy (produced by the linear mechanisms) in the wavenumber space repopulating transiently growing perturbations.

III. MAGNETIC FLUCTUATIONS IN THE LAPD SHEARED FLOW EXPERIMENTS

This work focuses on the LAPD experiments with a step-down electric field profile with velocity and density scale length of comparable sizes, in which magnetic fluctuations appear. Measurements of this regime using a vorticity probe have been reported in Ref. 11 and studied with reduced two-fluid model simulations. However, so far only measurements of density, temperature, and potential have been ana-
lyzed and modeled. Here, new measurements of magnetic field fluctuations are reported. Although magnetic fluctuations in the plane perpendicular to the external field are low, of the order $\delta B/B_0 \sim 10^{-5}$, there is a clear increase in the magnetic fluctuation levels in the shear layer region, which indicates that they may be driven by the drift wave-shear flow system.

Diagnostics of this experiment were made using a triple probe to measure density temperature and floating potential, a magnetic probe that measures all three components of the flow system. Experiments show the steady state background profiles of density, temperature, and plasma potential during the bias of density, temperature, and plasma potential, the electric field profile, obtained from the analysis of the generation of the magnetic fluctuations in the plane perpendicular to the external field are low, of the order $\delta B/B_0 \sim 10^{-5}$, there is a clear increase in the magnetic fluctuation levels in the shear layer region, which indicates that they may be driven by the drift wave-shear flow system.

Figure 1 shows a significant increase in magnetic fluctuations in the shear flow region. Sections IV and V present the analysis of the generation of the magnetic fluctuations in this DW-ZF system.

IV. BASIC EQUATIONS AND MATHEMATICAL FORMALISM

LAPD has a cylindrical geometry. The axial background magnetic field $B_0$ will be taken to lie along the $z$-direction. The sheared $E \times B$ flow region is located some distance $r = r_0$ from the main axis of the device. From Fig. 1, it follows that the size of the ZF ($L_E = 5$ cm) is much less than $r_0(=25$ cm). Therefore, the dynamical processes may be studied in the local Cartesian frame at the particular point $r = (r_0, 0, 0)$ with the $x$-axis directed along the radius and the $y$-axis along the azimuth. The set of model equations that describe the linear dynamics of perturbations on LAPD experiments is derived in Appendix A [see Eqs. (A17), (A22), and (A26)]. The equations are linear as the generation of Alfvénic-like fluctuations in the experiment takes place due to the linear mode coupling. In a regime with no dissipation, the set may be rewritten as

$$
\frac{d}{dt} \left( \nabla^2 \phi \right) - \frac{\partial v_{0,\phi}(x)}{\partial x} \frac{\partial \phi}{\partial y} = \frac{v_0^2}{c} \frac{\partial}{\partial z} \left( \nabla^2 \phi \right),
$$

$$
\frac{k_B T_e}{e} \frac{d}{dt} \left( \frac{n}{n_0} \right) + v_{de} \frac{\partial \phi}{\partial y} = \frac{v_0^2}{c} \frac{\partial}{\partial z} \left( \nabla^2 \phi \right),
$$

$$
\frac{d}{dt} \left[ (1 - \delta_e \nabla^2) \phi \right] = c \frac{\partial}{\partial z} \left( \phi - \frac{k_B T_e}{e} \frac{n}{n_0} \right) - v_{de} \frac{\partial \phi}{\partial y},
$$

where

$$
\frac{d}{dt} \frac{\partial}{\partial t} + v_{0,\phi}(x) \frac{\partial \phi}{\partial y}
$$

and $\phi$, $-\phi$, and $n$ are perturbations of the scalar electrostatic potential, the axial vector potential, and the density, respectively. The corresponding ZF velocity, magnetic field, and density are

$$
\frac{c}{B} (e_z \times \nabla \phi) = v_{0,\phi}(x) e_z + \frac{c}{B} (e_z \times \nabla \phi),
$$

$$
B = B_0 e_x + e_z \times \nabla \phi,
$$

$$
n = n_0(x) + \bar{n}.
$$

The other quantities used are the electron temperature and charge, $T_e$ and $e$; the electron drift velocity,
The wavenumbers of the SFH (Kelvin modes) vary in time along the flow shear. In the linear approximation, SFH “drift” in the \( \mathbf{K} \)-space (in wavenumber space).

Substitution of Eqs. (13) and (14) into Eqs. (10)–(12) results in

\[
K_\parallel (\tau) \frac{\partial \phi_h}{\partial \tau} + 2SK_\perp (\tau) \phi_h = iK_\perp (\tau) \psi_h, \tag{15}
\]

\[
\frac{\partial \eta_h}{\partial \tau} + iK_y \psi_h = -iV_{Te} \frac{\delta \phi}{\delta Z} (V_{de} \psi_h), \tag{16}
\]

\[
\frac{[1 + \delta^2 K_\perp^2 (\tau)]^{\frac{1}{2}}}{\partial \tau} \frac{\partial \psi_h}{\delta \tau} - 2S \delta^2 \phi \psi_h + i \phi_h - i \eta_h - iK_y \psi_h, \tag{17}
\]

where

\[
K_\parallel^2 (\tau) = K_\parallel^2 (\tau) + K_y^2. \tag{18}
\]

This system of equations corresponds to spectrally stable DWs. In fact, in accordance to Ref. 53, low frequency DWs are subject to the trapped-particle instability. To account for such instability, Eq. (16) becomes

\[
\frac{\partial \eta_h}{\partial \tau} + iK_y V_{de} (1 + i \epsilon) \psi_h = -iV_{Te} \frac{\delta \phi}{\delta Z} (V_{de} \psi_h), \tag{19}
\]

where \( \epsilon \ll 1 \). We do not fix a concrete value of \( \epsilon \), as the dynamical timescale is considerably shorter than the characteristic timescale of the trapped-particle instability in all cases (see below). Although the \( \epsilon \) term is formally inserted in the system, in fact, the trapped-particle instability has no significant influence on the DW dynamics.

In the simulations below, the quadratic form (spectral energy density) for a separate SFH as a measure of its intensity is

\[
E(\tau) = E_{kin}(\tau) + E_{kin}(\tau) + E_n(\tau) + E_m(\tau), \tag{20}
\]

with widely used definitions (cf. Ref. 54) of the energies that satisfy conservation of energy in the absence of shear flow and dissipation. The perpendicular and parallel kinetic, electron thermal, and magnetic energy spectral densities of the perturbations, respectively, are

\[
E_{kin}(\tau) = \hat{\rho}_e^2 \hat{K}_\perp^2 (\tau) |\phi_h|^2; \tag{21}
\]

\[
E_{kin}(\tau) = \hat{\rho}_e^2 \hat{K}_\perp^2 (\tau) |\psi_h|^2, \tag{22}
\]

\[
E_n(\tau) = |n_h|^2, \tag{23}
\]

\[
E_m(\tau) = \hat{\rho}_e^2 \hat{K}_\perp^2 (\tau) |\psi_h|^2. \tag{24}
\]

A detailed study of how increased levels of the biased electric field change the particle flux and fluctuation spectrum is reported by Carter and Maggs.\textsuperscript{12} Of particular relevance to the present work is their measurement, showing the stretching of the two-dimensional cross-correlation function in the direction of the sheared flow with increasing bias levels. In the present analysis, this stretching physics is contained in the wavenumber vector \( \mathbf{K}(\tau) \) time dependence induced by the shear flow parameter \( S \). The effect is relatively easy to understand: convection of the initial structures stretches them in the direction of the sheared flow. This oc-
and imaginary parts of the dispersive curves, respectively.

A. Perturbation spectrum in the shearless limit

The dispersion equation of our system may be obtained in the shearless limit ($S=0$) using the full Fourier expansion of the variables, including time. Although the roots of the dispersion equation obtained in the shearless limit do not adequately describe the mode behavior in the shear case, we use this limit to understand the basic spectrum of the considered system. Hence, using Fourier expansion of the field vector [e.g., $n_k(\tau) = \exp(-i\omega\tau)$], we derive for the shearless limit the cubic dispersion relation,

$$
\omega^3 [1 + \delta_k \hat{\omega}_0^2 (K_x^2 + K_y^2)] - \omega^2 K_y V_{de} - \omega [1 + V_{Te}^2 \delta_k \hat{\omega}_0^2 (K_x^2 + K_y^2)] + (1 + i\epsilon) K_y V_{de} = 0.
$$

(25)

This third order dispersion equation describes three different modes of perturbations: two high frequency kinetic Alfvén waves and a low frequency DW. Due to the nonzero electron skin depth scale ($\delta_k \neq 0$), the Alfvénic-like fluctuations are dispersive with \omega dependent on $K_x$. This fact is very important for mode coupling since $K_x$ is time dependent in nonuniform flow, which, in turn, makes \omega also time dependent.

The dispersion equation (25) is solved numerically for the parameters shown in Table I (taking $S=0$) and the real and imaginary parts of the dispersive curves, respectively, are plotted ($\omega_\alpha = \omega_\alpha(K_x)$, $\omega_\beta = \omega_\beta(K_x)$) in Fig. 3. The plots show that the magnitude of the frequencies of Alfvénic-like fluctuations differ substantially from the DW frequency for all values of $K_x$. Consequently, the Alfvénic-like and DWs are linearly coupled solely by the second/nonresonant channel at sizeable shear flow rates in Sec. II.

Figure 3 shows that maximum values of frequency ($\omega_{2\alpha}$) and growth rate ($\omega_{2\beta}$) for the least stable DW mode are 0.9 and 0.09, respectively, and are achieved at $|K_x/K_y| \leq 1$. Thus, $\omega_{2\beta} \approx S = 1$ and the trapped-particle instability has no significant influence on the dynamical phenomena.

According to Eq. (15) in the shearless limit, at $\omega_{2\beta} = 1$, when the axial vector potential is comparable to the electrostatic potential (i.e., at $|K_x/K_y| \leq 1$), DWs have a small electromagnetic component that arises from the parallel plasma current from the finite $en_x E_z = -k_B T_e n_e$. However, as $|K_x/K_y|$, $\omega_{2\beta} \rightarrow 0$, DWs become electrostatic. As to the other two modes, they are Alfvénic-like with high frequency $(\omega_{2\alpha}^{(c)}, \omega_{2\beta}^{(c)}) > 1$ and have dominant magnetic fluctuations for all $K_x$. Next we will analyze the coupling of the DW mode with these Alfvénic-like modes.

V. QUANTITATIVE ANALYSIS OF THE LINEAR DYNAMICS: TRANSIENT GROWTH AND MODE COUPLING

Spectral Fourier harmonics dynamics are studied by numerically solving the three complex time evolution equations (15), (17), and (19). Separating the fields into the real and imaginary parts with

$$
\phi_k = \phi_k^r + i\phi_k^i, \quad n_k = n_k^r + in_k^i, \quad \psi_k = \psi_k^r + i\psi_k^i
$$

(26)

gives the six real coefficient differential equations,

$$
K_1^2 (\tau) \frac{\partial \phi_k}{\partial \tau} - 2SK_1(\tau)K_y \phi_k^r = -K_1^2(\tau) \phi_k^r,
$$

(27)

$$
\frac{\partial n_k^r}{\partial \tau} - K_y V_{de} \phi_k^r - eK_y V_{de} \phi_k^i = V_{Te}^2 \delta_k K_1^2(\tau) \psi_k^r,
$$

(28)

$$
[1 + \delta_k \hat{\omega}_0^2 (\tau)] \frac{\partial \psi_k^r}{\partial \tau} = 2S \delta_k K_1(\tau) K_y \psi_k^r = -n_k^r + K_y V_{de} \psi_k^i,
$$

(29)

$$
K_1^2 (\tau) \frac{\partial \psi_k^r}{\partial \tau} - 2SK_1(\tau)K_y \psi_k^r = K_1^2(\tau) \psi_k^r,
$$

(30)

$$
\frac{\partial n_k^i}{\partial \tau} + K_y V_{de} \phi_k^i - eK_y V_{de} \phi_k^r = -V_{Te}^2 \delta_k K_1^2(\tau) \psi_k^r,
$$

(31)
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The initial conditions that correspond to the pure DW SFH are presented in Figs. 4–8. Recall that the action of the flow shear on the dynamics of a perturbation SFH. The character of the dynamics of relations \(K_x, K_y\) depends on which mode SFH is initially imposed in the dynamics of the system. The numerical simulations are performed using the MATHEMATICA numerical ordinary differential equation solver, an implementation of the stiff Gear backward differentiation method. Note that the action of the flow shear on the dynamics of DW SFH at wavenumbers \(K_x(0)/K_y \gg 1\) is negligible. This fact permits writing the initial conditions for the pure DW SFH (see Appendix B).

The simulations reveal a novel linear effect—the excitation of Alfvénic-like fluctuations—that accompanies the linear evolution of DW mode perturbations in the ZF. (For a physical description of this excitation refer to Refs. 32, 34, and 35.) Mathematically, the problem is equivalent to time-dependent scattering theory in quantum mechanics. By using a \(3 \times 3\) matrix representation of the system, formal solutions can be written in terms of the time ordering operator and exponentials of matrices.

The evolution of the initial DW SFH according to the dynamic equations (27)–(32) for the LAPD parameters in Table I is presented in Figs. 4–8. Recall that \(K_x(\tau)\) changes in time according to Eq. (14): the shear flow sweeps \(K_x(\tau)\) to low values and then back to high values but with negative \(K_x(\tau)/K_y\). As shown in graph (a) of Fig. 4, while \(K_x(\tau)/K_y > 1\), the DW SFH undergoes substantial transient growth without any oscillations and the magnetic fluctuations are small (\(|\psi_0|/\phi_0 < 1\)). Significant magnetic field fluctuations (\(|\psi_0|/\phi_0 \approx 1\)) appear when \(K_x(\tau)/K_y \approx 1\). While \(K_x(\tau)/K_y < 0\), the DW SFH generates the related SFH of Alfvénic-like wave modes through the second channel of the mode coupling outlined in Sec. II. This generation of Alfvénic-like wave modes is especially prominent in Fig. 5, where significantly higher frequency oscillations of all the fields are clearly seen at times when \(K_x(\tau)/K_y < 0\). Figure 6 shows the related dynamics of the different energies. It indicates a substantial transient burst of the electron thermal energy (elec-
tron density) of fluctuations and an appearance of Alfvénic-like fluctuations.

Comparing Figs. 6 and 7(a), one can see that the dynamics only slightly vary with variation in $\epsilon$: the amplification is just twice as strong at $\epsilon=0.1$ than at $\epsilon=0$. Consequently, the transient burst substantially exceeds the growth of the classical dissipative trapped-particle instability of the system.53

Thus, the growth is governed by the flow non-normality due to the shear flow as described at the end of Sec. IV. The Alfvénic-like fluctuation generation is intensified with increasing $K_y$, cf. Figs. 6 and 7(b), i.e., the generation increases with scaling down of SFHs in the zonal direction.

Increasing $V_{Te}^2$ changes the energy evolution [cf. Figs. 6 and 7(c)]: the total energy growth increases due to an increase in the electron thermal energy transient growth. However, the magnetic energy growth somewhat decreases.

The phenomenon strongly depends on the value of the normalized shear parameter $S$ [cf. Figs. 6 and 7(d)]: the Alfvénic fluctuation generation becomes appreciable at $S=0.3$, pronounced at $S>0.5$ and dominant at $S=1$, such as in the LAPD experiment ($V_{pe}=2 \times 10^{-3}$; $\delta_0=5 \times 10^{-3}$; $V_{Te}^2=2$; $S=1$). Note that simulation 5 starts with $K_y(0)=0.9 \times 10^4$ to retain the same dynamical time as in the previous simulations $\tau_{dy}=K_y(0)/K_y S$.

The curves in Fig. 8 compare the perpendicular kinetic energy transient growth according to Eq. (15) to that of the pure vortex mode [i.e., when the right-hand side of Eq. (15) equals zero]: the pure vortex (solid line on the graph) undergoes transient growth, while in the considered case, the dynamics of the perpendicular kinetic energy (dashed line on the graph) are complicated and reduced due to the action of other fields.

VI. CONCLUSIONS AND DISCUSSION

Drift-Alfvén waves are investigated in plasma with a significant level of background sheared flow. Magnetically confined plasmas in both laboratory experiments, space physics, and coronal loops are examples where sheared flows occur. As part of the LAPD basic physics experiments, detailed measurements of the fluctuating plasma density, electrostatic potential, and magnetic fields were carried out. We have been guided by the data from the LAPD (see Sec. III) experiments in developing the theory given in this work.
We show that the linear dynamics of DWs, such as those in the LAPD experiments, are qualitatively changed by the presence of sheared flows when the shear normalized parameter $S$ [see Eq. (1)] approaches unity and, consequently, there is strong excitation of magnetic fluctuations by the drift wave–shear flow system. The shear flow induces transient growth/bursts along with complex temporal wave forms and generates Alfvénic-like fluctuations from DWs. We show that the trapped-particle or any classical DW instability is far slower than these transient bursts and has no notable influence on the dynamic processes for the parameter values of the LAPD experiments. The frequency of the bursts is determined by the frequency of the generated Alfvénic-like waves [cf. Figs. 5 and 7(b)]. In the case presented by Cooper, where the flow is parallel to the background magnetic field, the frequency of the bursts depends only on the value of velocity shear parameter.

The complex linear dynamics are a result of the shear flow continually sweeping the wavenumber of the DW SFH $K_x(\tau)$ to low values and then back to high values. In this time-dependent sweeping of $K_x(\tau)$, the SFH undergoes substantial transient growth and, when $K_x(\tau)/K_x<0$, it generates the related SFH of Alfvénic-like wave modes, as explained in Sec. V and illustrated in Figs. 4 and 5. The linear mode coupling channel at work is nonresonant (see Refs. 32 and 35 for details) and universally leads to energy exchange between different perturbation modes at high shear rates. This unambiguously occurs in the LAPD experiments with $S \approx 1$ and may account entirely for the observed magnetic fluctuations. Flow non-normality induced mode coupling should also occur in tokamaks and may be related to the new phenomenon.

In space physics the effect could be associated with sheared Earthward flows in the nightside plasma sheet that are driven by solar wind with southward embedded solar magnetic field components. Spacecraft in the plasma sheet measure high speed sheared flows driven by the convective electric field. Enhanced magnetic fluctuations associated with these flows are also observed. It remains to make a quantitative analysis of the magnetospheric problem. Finally, note that nonlinear simulations showing the growth of vortex structures out of the linear transients are reported in Ref. 57. Further nonlinear studies are being planned for the laboratory and space physics settings of this qualitatively new phenomenon.

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APPENDIX A: DERIVATION OF REDUCED PLASMA EQUATIONS: IN CGS

We derive the set of model equations that describes the linear dynamics of perturbations on LAPD experiments where the generation of Alfvénic-like fluctuations in DW-ZF systems is observed. This set of three model equations are written for perturbations of the scalar electrostatic potential $\tilde{\phi}$, the component of the electromagnetic vector potential parallel to the mean magnetic field $-\tilde{\psi}$, and the density $\tilde{n}$. Therefore, we express all physical quantities via $\tilde{\phi}, \tilde{\psi}, \tilde{n}$ and linearize the equations in parallel.

The electrostatic and magnetic potentials are represented by $\phi = \phi_0(x) + \tilde{\phi}$ and $\psi = \psi_0(x) + \tilde{\psi}$, and the fields as

$$E_x = -\nabla_x \phi, \quad \psi = \psi_0 + \tilde{\psi},$$

$$E_i = -\tilde{b} \cdot \nabla \phi + \frac{1}{c} \frac{\partial \tilde{\psi}}{\partial t},$$

$$B = B_0 \tilde{e}_x + \tilde{\phi} = B_0 \tilde{e}_x + \tilde{e}_x \times \nabla \tilde{\psi},$$

where $\phi_0(x)$ is the mean electrostatic potential and $B_0$ is the value of the background magnetic field.

The cross electron and ion drift velocities have the form

$$v_{e\perp} = v_E + v_{de}, \quad v_{i\perp} = v_E + v_{pi},$$

where $v_E$ and $v_{0i}$ are full and mean sheared $E \times B$ flow velocities.

$$v_E = \frac{e}{B_0^2} \frac{E_x \times B}{B_0} = \frac{e}{B_0} (\tilde{e}_x \times \nabla \tilde{\phi}),$$

$$v_{0i} = \frac{e}{B_0} (\tilde{e}_x \times \nabla \tilde{\psi})_0 = \frac{e}{B_0} \frac{\partial \phi_0}{\partial x} \tilde{e}_y,$$

$$v_{de} = -\frac{e}{en_0 B_0} \nabla p_e = -\frac{ck_T}{en_0 B_0} \frac{1}{n_0} \frac{\partial n_0}{\partial x} \tilde{e}_y,$$

$$v_{pi} = \frac{m_e c^2}{\omega_{pe} B_0} \frac{d E_x}{dt} = -\frac{m_e c^2}{e B_0} \frac{d}{dt} \frac{\partial \phi}{\partial \tilde{y}},$$

$$n = n_0(x) + \tilde{n}, \quad p_e, \quad \text{and} \quad T_e$$

are the electron density, pressure, and temperature, and $L_n$ is the ZF radial size. The basic ZF ve-
locity $v_0$, is directed along the $y$-axis and defines the convective time derivative,
\[ \frac{d}{dt} = \frac{\partial}{\partial t} + v_0 \cdot \nabla. \quad (A9) \]

In Alfvén DW turbulence, there are two important nonlinear directional derivatives given by
\[ \mathbf{\delta B} \cdot \nabla f = \frac{1}{B_0} (\mathbf{e}_z \times \nabla \tilde{\psi}) \cdot \nabla f = \frac{1}{B_0} [\psi, f], \quad (A10) \]

\[ \mathbf{v}_E \cdot \nabla f = \frac{c}{B_0} \frac{\partial \phi_0}{\partial x} \mathbf{e}_x + \frac{c}{B_0} (\mathbf{e}_x \times \nabla \tilde{\phi}), \quad (A11) \]

\[ = v_{0y} \partial_y \mathbf{e}_x + \frac{c}{B_0} \psi f]. \quad (A12) \]

These directional derivatives are Lie derivatives along the dynamical vector fields. The derivatives are strongly nonlinear when the fluctuation amplitudes reach the levels $\mathbf{\delta B}/B_0 = |k_\perp|/k_\perp$ and $\mathbf{v}_E = |\omega|/k_\perp$. In the nonlinear vortices, both nonlinearities reach their limiting values, which are large for the lowest values of $k_\perp$.

The first equation is derived from the charge conservation equation $\partial (q, n, q, n, v, \mathbf{v})/\partial t + \nabla \cdot (q, n, v, n, \mathbf{v}) = 0$ for a quasineutral inviscid incompressible fluid
\[ \nabla \cdot [en(v_0 - v)] = 0, \quad (A13) \]

\[ \nabla \cdot J_\parallel = - \nabla \cdot J_\perp. \quad (A14) \]

Taking into account the axial component of Ampère’s law ($\nabla \times \mathbf{A} = 4\pi J_\parallel/c$) and the parallel electron current expression ($J_\parallel = env_\parallel$), one can express the parallel electron velocity via $\tilde{\psi}$,
\[ v_\parallel = - \frac{c}{4\pi en} \nabla \tilde{\psi}. \quad (A15) \]

The perpendicular current is supplied by the ion polarization drift,
\[ \mathbf{J}_\perp = env_\parallel = - \frac{nm_e e^2 (\partial/\partial t + v_0 \partial/\partial y)}{B^2} \nabla \tilde{\psi}. \quad (A16) \]

Substituting expressions of $\mathbf{J}_\parallel$ and $\mathbf{J}_\perp$ from Eqs. (A15) and (A16) into Eq. (A17), one obtains
\[ \left( \frac{\partial}{\partial t} + v_{0y} \frac{\partial}{\partial y} \right) \nabla \tilde{\psi} = \frac{v_0}{c} \frac{\partial^2}{\partial z^2} (\nabla \tilde{\psi}), \quad (A17) \]

where $v_0 = B_0/\sqrt{4\pi n_0 m_e}$ is the Alfvén velocity.

The second equation is derived from the electron continuity equation,
\[ \frac{\partial n_e}{\partial t} + \nabla (n_e \mathbf{v}_e) = 0 \quad (A18) \]

or
\[ \frac{\partial \tilde{n}_e}{\partial t} + \mathbf{v}_e \cdot \nabla (n_0(x) + \tilde{n}) = -(n_0(x) + \tilde{n})(\nabla \cdot \mathbf{v}_e). \quad (A19) \]

Taking into account Eqs. (A4)–(A7) and (A15), in the linear regime, the second terms on the left-hand side and the right-hand side of Eq. (A19) become
\[ \mathbf{v}_e \cdot \nabla [n_0(x) + \tilde{n}] = v_{0y} \frac{\partial n_0}{\partial y} - \frac{c}{B} \frac{\partial \tilde{\psi}}{\partial x}, \quad (A20) \]

\[ -[n_0(x) + \tilde{n}] (\nabla \cdot \mathbf{v}_e) = -n_0(x) \frac{\partial n_0}{\partial z} = \frac{c}{4\pi en_0} \nabla^2 \tilde{\psi}. \quad (A21) \]

Introducing the density inhomogeneity scale length $L_n = n/|\partial n_0/\partial x|$, finally, the second equation reduces to
\[ \frac{\partial}{\partial t} + v_{0y} \frac{\partial}{\partial y} \tilde{\psi} + \frac{c}{4\pi en_0} \nabla^2 \tilde{\psi} = \frac{c}{\partial t} \frac{\partial}{\partial y} \tilde{\psi}. \quad (A22) \]

The third equation is derived from the linearized electron parallel momentum equation in the absence of resistivity,
\[ m_e \left( \frac{\partial}{\partial t} + v_{0y} \frac{\partial}{\partial y} \right) \tilde{v}_e = - \frac{en}{c} E_1 - v_{0y} \breve{\nabla} \tilde{\psi}. \quad (A23) \]

From Eqs. (A1) and (A10) in the linear limit follows
\[ E_1 = - \frac{\partial \tilde{\psi}}{\partial t} - \frac{1}{B_0} \frac{\partial \tilde{\psi}}{\partial y} \frac{c}{\partial t} = - \frac{\partial \tilde{\psi}}{\partial t} + \frac{1}{c} \left( \frac{\partial}{\partial t} + v_{0y} \frac{\partial}{\partial y} \right) \tilde{\psi}, \quad (A24) \]

\[ \nabla \cdot \mathbf{p}_e = k_B T_e \nabla n_e = k_B T_e \left( \frac{\partial \tilde{n}}{\partial z} + \frac{1}{B_0} \frac{\partial n_0}{\partial y} \right) \frac{1}{\partial t} \frac{\partial n_0}{\partial y} \right) \tilde{\psi} = n_0 \left( k_B T_e \frac{\partial \tilde{n}}{\partial z} + \frac{v_0}{c} \frac{\partial \tilde{\psi}}{\partial y} \right). \quad (A25) \]

Taking into account Eqs. (A15), (A24), and (A25), Eq. (A23) becomes
\[ (1 - \delta^2 / \omega^2) \left( \frac{\partial}{\partial t} + v_{0y} \frac{\partial}{\partial y} \right) \tilde{\psi} = \frac{c}{\partial z} \left( \frac{\partial}{\partial t} + \frac{k_B T_e}{c} \frac{\partial \tilde{n}}{\partial n_0} \right) - v_{0e} \frac{\partial \tilde{\psi}}{\partial y}, \quad (A26) \]

where $\delta^2 = \omega^2 / \omega^2$.

**APPENDIX B: INITIAL CONDITIONS**

Consider a SFH for which $K_s(0)/K_s > 1$ and, thus, the action of the shear $S$ is negligible. In this case, the instability term ($\epsilon = 0$) can be neglected. Assuming
\[ \begin{pmatrix} \phi_k(	au) \\ n_k(	au) \\ \psi_k(	au) \end{pmatrix} = \begin{pmatrix} \phi_k \\ n_k \exp(-i\omega_k \tau) \\ \psi_k \end{pmatrix}, \quad (B1) \]

Eqs. (15)–(17) become
\[ -\omega_k [K_s(0), K_s] \phi_k = \psi_k, \quad (B2) \]

\[ -\omega_k [K_s(0), K_s] n_k + K_s V_de \phi_k = -V_e^2 \tilde{\xi}_k^2 K_s K_s^2 \psi_k, \quad (B3) \]
\[
- \{ 1 + \delta^2 [K_x^2(0) + K_y^2] \} \omega_0 \omega_0[k_x(0), k_y] \psi_k = \phi_k - n_k V_d \psi_k \\
\text{in terms of } \phi_k, \ n_k, \text{ and } \psi_k \text{ are}
\]
\[
n_k = \left[ \frac{K_x V_d}{\omega_0} - V_T^2 \delta \delta[K_x^2(0) + K_y^2] \right] \phi_k, \\
\psi_k = - \omega_0 \omega_0[k_x(0), k_y] \psi_k. 
\]

Solving the dispersion equation (25) at \( K_x = K_y(0) \) and defining the DW frequency (i.e., the smallest \( \omega_0 \)) as \( \omega_0[k_x(0), K_y] = \omega_0, \) Eqs. (B5) and (B6) take the form
\[
n_k = \left[ K_x V_d \omega_0 - V_T^2 \delta \delta[K_x^2(0) + K_y^2] \right] \phi_k, \\
\psi_k = - \omega_0 \phi_k. 
\]

At \( K_x(0)/K_y \gg 1, \) \( \omega_0 \ll 1 \) and, consequently, \( \psi_k \ll \phi_k \).

Finally, taking into account Eq. (26), one can write initial conditions for numerical solutions of Eqs. (27)-(32) as
\[
\phi_k'' = 0, \quad \psi_k' = 1, \\
n_k'' = 0, \quad n_k' = \frac{K_x V_d}{\omega_0} - V_T^2 \delta \delta[K_x^2(0) + K_y^2], \\
\phi_k' = 0, \quad \psi_k' = - \omega_0. 
\]