A three-dimensional experimental study of lower hybrid wave interactions with field-aligned density depletions

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Abstract

Rocket observations of spatially localized bursts of intense lower hybrid wave radiation have been made for decades. As the ability of rocket observations to measure small-scale structure increases, scientists are better able to understand the physics responsible for the observations. However, rocket measurements are limited in their ability to fully diagnose the phenomenon they seek to measure. Therefore a laboratory experiment was undertaken at the Large Plasma Device (LAPD) at UCLA to study the interaction of lower hybrid waves with a field-aligned density depletion. The laboratory parameters were chosen to mimic the physics in the ionosphere. Laboratory experiments allow one not only to study the effect of different interaction parameters, like wave frequency or striation size, but also to fully diagnose the wave fields in three spatial dimensions and time. These experiments have shown that large amplitude wave fields are localized to the areas of largest density gradient. The field pattern is independent of striation size and shape. The experimental results relate strongly to space-based observations. The laboratory results, therefore, can act as a guidepost to assist the understanding of the more complex and difficult-to-understand plasmas encountered in the ionosphere.
Figure 1

Figure 2
Figure 3

Figure 4
Figure 5
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Introduction

The topic of wave interactions with plasma non-uniformities is a rich field which is now under investigation in the laboratory. The natural evolution of laboratory experiments to study these interactions should hold interest for those in the space-physics community. Space, ionospheric, and magnetospheric plasmas are non-uniform in some way. A well-designed laboratory experiment has the potential to provide measurements in detail far greater than can currently be obtained by in-situ measurements. Such detail can provide new insight into the physical mechanisms involved and can help direct the development of theories to explain the space-based observations.

The observation of lower hybrid wave interactions with a field-aligned density depletion by sounding rockets in the ionosphere provided the inspiration for a series of laboratory experiments at the LAPD at UCLA. These experiments were designed so that their results would not only provide information about the wave/plasma non-uniformity interaction in general, but also would provide information about the ionospheric observations.

Lower hybrid wave radiation has been observed in the ionosphere for decades [Barrington et al., 1971; Hoffman and Laaspere, 1972 for example]. It is characterized by broadband electrostatic noise above a low frequency cut off (the lower hybrid resonance frequency) and is often called VLF hiss. The source of this hiss was described by Maggs [1976, 1978, 1989]. His theories have largely been confirmed by observation [Vago et al, 1992 for example]. This background hiss, which rains down on the ionosphere, provides the source for lower hybrid wave radiation throughout the measurement region for satellites and rockets. As Maggs [1989] predicted, waves at lower altitudes are more electrostatic, as their angle of propagation with respect to the background field becomes more orthogonal.
Intense localized bursts of lower hybrid wave radiation were first observed in the ionosphere by the MARIE sounding rocket [LaBelle, 1986]. These bursts differ from the background hiss in that they are short-lived and have amplitudes significantly larger than the background. At the time, LaBelle et al. were not able to make any statement about the relationship of these bursts, which they called “spikelets,” to changes in the plasma density. In fact, while they believed that these spikelets could be stationary structures, they could not discount the possibility that the structures moved at the ion drift velocity. They were, however, able to demonstrate a correlation between the spikelets and transverse ion acceleration. Ions accelerated at the rocket altitudes (450 - 650 km on MARIE; 800 - 1000 on TOPAZ III) would explain ion conics observed by satellites at higher altitudes [Klumpar, 1979, for example] and were therefore of significant interest to the space community. Later observations confirmed the correlation between the spikelets and transversely accelerated ions (TAI): Garbe et al. [1992] from TOPAZ II (TOpside Probe of the Auroral Zone), Vago et al. [1992] from TOPAZ II and III; Kintner et al. [1992] from TOPAZ III; and Arnoldy [1992] et al. from TOPAZ III.

The observations made by TOPAZ III included some new information not seen by MARIE or TOPAZ II. Arnoldy et al. [1992] and Vago et al. [1992] were able to demonstrate a correlation between the spikelets and density depletions. Vago et al. calculate from a Langmuir probe held at a fixed bias that the density corresponding to the occurrence of lower hybrid spikelets was between 20% and 80% of the background density. Though interpretations of density from rocket-born Langmuir probes is difficult, and Vago’s density measurements have been called into question, Vago was able to demonstrate that density depletions are an important part of lower hybrid spikelets. Furthermore, Vago et al. confirmed that the observed spikelets were lower hybrid mode noise by performing an interferometric analysis of their data. Lower hybrid waves in conjunction with density striations have also been observed by satellites, and in general
the density perturbations viewed at the higher altitudes satellite frequent are smaller. A statistical survey [Heymork et al, 2000] of perturbations encountered by the Freja satellite travelling at 1450-1750 km altitudes (rockets go up to about 1000 km) showed that the density cavities have a Gaussian shape at right angles to the background magnetic field and are usually less than ten percent deep.

Several questions about lower hybrid spikelets remained unanswered. Did lower hybrid hiss impinge on pre-existing striations in the ionosphere? Or did the density depletion arrive as a result of a non-linear wave interaction?

Given the difficulties with theories that required lower hybrid wave-created density depletions, attention focused on the idea that lower hybrid noise impinged on pre-existing density depletions. The change in emphasis to examine interactions with pre-existing striations, has been accompanied by a shift in emphasis away from investigations of ion acceleration and toward the structure associated with such an interaction for the time being. For example, while Andrulis et al. [1996] postulate that small amplitude density cavities can transfer energy to short-wavelength modes which then transfer energy to ions, they focus much of their efforts in the evolution of the wave potential structure.

Work by Schuck et al. [1998] presents a fully-developed theory for the structure of the intense lower hybrid bursts (now re-termed Lower Hybrid Solitary Structures or LHSS). This electrostatic, fluid theory, predicts a continuous spectrum of eigenfrequencies at $\omega > \omega_{lh}$ which rotate in a clockwise sense. They compare the theory to TOPAZ III observations with good agreement. Observations and analysis by Bonnell et al. [1998] for the PHAZE II (PHysics of Auroral Zone Electrons) sounding rocket and Pin on et al. [1998] for the AMICIST (Auroral Microphysics and Ion Conics Investigation of Space and Time) rocket also claim good agreement with Schuck et al.
Other analysis by McAdams et al. [1998] of the PHAZE II data shows that the most intense bursts of lower hybrid radiation occur at the steepest density gradients. They conclude from their analysis that the structure measured by the rocket must come from lower hybrid hiss impinging on a pre-existing depletion. Furthermore, observations by Delory et al [1997] show intense lower hybrid bursts associated with wave formations as short as the smallest lengths measurable by their instrument, the University of California at Berkeley Alaska 93 auroral sounding rocket. These bursts were associated with sudden changes in measured plasma density. They measure no correlation with significant ion heating.

Qualitatively similar measurements have been made at higher altitudes by satellites. Freja measurements, for example, reported by Eriksson et al. [1994] show a direct correlation between density depletions and intense bursts of lower hybrid energy. FAST satellite measurements presented by Ergun [1998] present electric fields similar to those presented by Kintner [1992].

In short, lower hybrid solitary structures have been observed with a variety of rockets and satellites and under a number of conditions. The very nature of such measurements, however, preclude the possibility of measuring the entire spatial and temporal dependence of the electric field patterns. Making observations of the same physical process in the laboratory would provide an opportunity to measure fields through the entire region of interest.

Such observations were made as part of a series of experiments designed to study the interaction of lower hybrid waves with a field-aligned density depletion in the laboratory. The ability to measure three-dimensional wave fields as a function of time in a controlled setting makes the use of laboratory experiments ideal for studying the interaction. Furthermore, laboratory study provides the opportunity to examine the effects of wave
frequency and striation size and shape on the interaction.

In order to make any comparison between data taken in space plasmas and data taken in the laboratory, the case must first be made that the physics which take place in the laboratory are applicable outside the vacuum chamber. Obviously one can not create an exact replica of ionospheric plasma in the laboratory. Laboratory plasmas, out of necessity, have relatively large magnetic fields and densities; time scales and scale lengths are relatively small. However, it is not necessary to create an exact replica of the ionospheric plasma to recreate the physics of the ionosphere. The physics is controlled by the ratios of key parameters. In the case of these experiments, the key parameters are the ratio of wave frequency to characteristic plasma frequencies; and the ratios of various scale lengths. These ratios, comparing typical values in the ionosphere and in the L\(\text{APD}\), are shown in Table One.

The values for the L\(\text{APD}\) compare favorably with those in the ionosphere. The ratio of \(f_{pe}/f_{ce}\) compares poorly. However, it has been shown that this ratio does not affect the propagation of lower hybrid waves in the ionosphere [Strangeway, 1997]. It is not a coincidence that the parameters coincide so well. The experiment was specifically designed to allow us to compare the results from the laboratory with those from the ionosphere. The experimental design is discussed in chapter three. After discussing the experiment and its results, we will return to ionospheric data and compare space-based observations with the results from the laboratory experiments.

**The Dispersion Relation**

In order to better understand the propagation of lower hybrid waves, it is convenient to turn to the dispersion relation for an infinite, cold, uniform, collisionless plasma. It may seem like this is an odd choice of dispersion curves to study. While it is not hard to
say that the plasma is collisionless (the wave frequency, $f >> \nu_{e,i}$, the plasma collision frequencies); and it is not too much of an exaggeration to call the plasma infinite, it seems strange to describe the plasma as cold and uniform. After all, the entire premise of these experiments is to describe a wave interaction with a non-uniformity in the plasma. Furthermore, the non-uniformity has a scale size (the gradient scale length) on the order of an ion gyro-orbit.

Fortunately, the degree to which the experimental plasma is not cold and uniform does not significantly affect the properties of the plasma as it relates to the dispersion of the waves parallel to the background field. The reasons are as follows: while the ion gyro-orbit has the same scale length as the non-uniformity, the gyro-frequency is much lower than the lower hybrid frequency. Therefore, to first order, the ion gyro-orbits are effectively unperturbed and one can treat the ions as if they are cold. The existence of the non-uniformity does not preclude the use of a uniform dispersion relation either. Since the non-uniformity is in the perpendicular direction only, the effects on the dispersion of the waves occur only in the perpendicular direction. The response of the plasma in the direction parallel to the background field is unaffected by the presence of the striation and the plasma continues to be fairly well described by the uniform plasma dispersion relation. This has been experimentally verified with parallel wavelength measurements.

By solving the wave equation for non-trivial $E$, one arrives at the Appleton-Hartree dispersion relation [Stix,1962].

\[ n^2 = 1 - \frac{2\Pi \omega^2 (1 - \Pi)}{2\omega^2 (1 - \Pi) - \omega_{ce}^2 \sin^2 \theta \pm \omega_{ce} \Delta} \]  \hspace{1cm} (1)

where

\[ \Delta = \left[ \omega_{ce}^2 \sin^4 \theta + 4\omega^2 (1 - \Pi)^2 \cos^2 \theta \right]^{\frac{1}{2}} \]  \hspace{1cm} (2)

and

\[ \Pi = \frac{\omega_{pe}^2 + \omega_{ni}^2}{\omega^2} \approx \frac{\omega_{pe}^2}{\omega^2}. \]  \hspace{1cm} (3)
It is possible to approximate the electrostatic portion of Equation (3) by assuming the angle of propagation is exactly $90^\circ$ and that wave magnetic fields do not come into play. The mathematics become much simpler and the dispersion relation reduces to

$$\varepsilon k^2 + \varepsilon k_{\|}^2 = 0 \quad (4)$$

Unfortunately, this simplified dispersion relation can not provide all the information one finds in the full relation. Therefore, we continue to use Equation (1) to find the wave properties.

Equation (1) gives rise to two roots in $n^2$; the positive root gives rise to the lower hybrid wave dispersion relation for waves whose frequency is between the lower hybrid frequency and the minimum of the electron plasma and electron cyclotron frequencies. The lower hybrid frequency is given by

$$\omega_{\text{LH}} \equiv \left(\frac{1}{\omega_{ci}^2 + \omega_{pi}^2} + \frac{1}{\omega_{ce} \omega_{ci}}\right)^{-\frac{1}{2}} \quad (5)$$

The lower hybrid frequency is dependent on both the plasma density and the background magnetic field. At high densities, $f_{\text{lh}}$ is dependent on the field; at low densities, $f_{\text{lh}}$ is density dependent. Figure 1 is a plot of the lower hybrid resonance frequency versus density for a number of magnetic fields. In the laboratory experiments we always launched waves above $f_{\text{lh}}$, (sometimes termed the lower hybrid resonance frequency) so that there was no lower hybrid resonance location in the plasma. This was intentional since at the resonance the wave ceases to propagate.

One can solve equation (1) numerically for a given value of $B_0$, wave frequency, $\omega$, density and ion species. Typically, one finds the locus of accessible $k_{\|}$ as a function of
k⊥. An example of this type of plot is found below in Figure 2 for a background field of 
$B_0 = 1650$ G, a density of $5 \times 10^{11}$ cm$^{-3}$ in argon for two different frequencies; $f_{\text{li}} = 15$ MHz.

By examining this curve, we can learn some useful information about the propagation 
of lower hybrid waves in an infinite, uniform, cold plasma. First, it is useful to examine 
some of the gross characteristics of this curve. Note the different scales for $k_\parallel$ and $k_\perp$. 
For large $k_\perp$ values ($k_\perp > 1$ for the curves in Figure 2), the k-vectors which lie on these 
curves are almost entirely perpendicular to the background field. The angle to which the 
k-vectors asymptote is the resonance cone angle; for these experiments, this angle is 
typically $88 - 89^\circ$. One should also be aware that the group velocity is perpendicular to 
this curve at every point [Poeverlein, 1948]. Therefore, the group velocity for curves 
with large $k_\perp$ is almost entirely parallel to the background field. In other words, the 
energy propagates down the field at the complement to the resonance cone angle.

The same dispersion curve describes not only the lower hybrid wave, but also the 
whistler wave. In fact, there is no sharp distinction between these two waves. Typically, 
the dividing line between them is taken to be the angle at which the $k_\parallel$ is a minimum. In 
Figure 2 this critical value is at about $k_\perp = 1$ cm$^{-1}$ for both curves. At higher values of $k_\perp$, 
one finds the lower hybrid waves; at lower values one finds the whistler waves.

As was stated earlier, there is no clear distinction between these two modes. However, one usually considers the whistler wave to be electromagnetic and the lower 
hybrid wave to be electrostatic. Such an approximation can be a useful way to characterize 
lower hybrid waves, but, it is an oversimplification. The division between whistler and 
lower hybrid waves, however, is not arbitrary. For lower hybrid waves, the k-vectors are 
very close to the resonance cone angle (almost perpendicular to $B_0$). Whistler waves can 
have k-vectors at any angle smaller than the resonance cone angle. Furthermore, while
the lower hybrid wave group velocity is almost entirely parallel to the background field, whistler waves have a maximum angle of propagation as high as 15° - 20°. The most dramatic difference between whistler and lower hybrid waves may be in the relative sign of the perpendicular phase velocity with respect to the perpendicular group velocity. As one can see in Figure 2, the group velocity has a negative perpendicular component while the phase velocity (which is in the same direction as the k-vector) has a positive perpendicular component. The lower hybrid wave's phase fronts appear to move backward across $B_0$, as was shown in the laboratory by Stenzel and Gekelman [1975] in which the dispersion relation was measured for the first time. Bellan and Porkolab [1975] made subsequent measurements and shortly afterwards Gekelman and Stenzel [1975] observed the ponderomotive effects of the wave. The lower hybrid wave is a backward-travelling wave in the perpendicular direction. The group and phase velocities for whistler waves have parallel (not anti-parallel) perpendicular components. The point which separates whistler from lower hybrid waves is, therefore, the minimum in the $k_\perp - k_\parallel$ curve shown in figure 2. A discussion of this dispersion in connection with VLF Hiss was made by Taylor and Shawan [1974] where they pointed out that the amplitudes of the electric and magnetic fields of these waves are highly dependent on the wave normal angle. As one approaches the resonance cone angle the magnetic field of the wave gets smaller. It is strictly zero at the resonance cone angle, but at that point the wave ceases to propagate. Finally both whistler and lower hybrid waves were studied in an experiment by Bamber et al [1995] by measuring the magnetic fields associated with them. These measurements were subsequently verified by direct measurement of the wave electric fields [Rosenberg and Gekelman, 1998].

The wave equation (whose non-trivial solution is Equation (1)) also provides information about the electric field amplitudes. Taken together with $\mathbf{B} = \mathbf{n} \times \mathbf{E}$, one can find the magnetic and electric fields and the electric and magnetic field wave energy as a
function of $k_z$. In fact, one finds that the amount of energy in the magnetic fields outside of the curve minimum — in what most would agree is the lower hybrid mode — can be significant. For the experiments described here, between 10 and 50%.

**Description of Experiment**

The experiments were conducted at the LAPD at UCLA. This device comprises a 10 m long vacuum chamber surrounded by 68 axially aligned magnet coils (see Figure 3). For these experiments, the magnets are set to provide a uniform 1650 G background field. The large field ensures that the lower hybrid waves are density dependent for the densities in this experiment (see Figure 1). The plasma is created by pulsing an oxide-coated cathode negative with respect to a grid anode 60 cm away. The electrons emitted by the cathode ionize the gas which has been backfilled into the chamber, argon in these experiments. The background density, as measured by Langmuir probes and a microwave interferometer, is $n = 5 \times 10^{11}$ cm$^{-3}$. Other plasma parameters are: $T_e = 5 \text{ eV}$, $T_i = 1 \text{ eV}$, $R_{ci} = .39 \text{ cm}$, $R_{ce} = 3.2 \times 10^{-3} \text{ cm}$. The experimental repetition rate is 1.1Hz. The experiment is performed in the afterglow of a DC discharge, several microseconds after turnoff. There are no primary (ionizing) electrons present at this time. The density and electron temperature do not vary over the ensuing several microseconds of experimental time. The lower hybrid resonant frequency for these conditions is $f_{lh} = 15$ MHz and is heavily density dependent.

The method by which the striation is created involves taking advantage of the method by which the plasma is created. If one blocks the ionizing electrons which leave the cathode, one effectively inhibits the formation of plasma along the field lines which are blocked. Thus, one can create striations of varying sizes and shapes by inserting a paddle between the cathode and anode. Three different striations were studied in these experiments; their characteristics will be shown in the next section. Each striation has an
asymmetry due to the fact that the paddle used to block primary electrons has a support which, though small, also blocks electrons.

In order to launch lower hybrid waves, a phased-array comprising four electrically isolated “plates” was built. Each “plate” consists of a copper wire running parallel to the background field with a spacing of 1 cm across the field. The copper wire spacing is small enough to provide an equipotential surface, but large enough to minimize the disruption to the plasma density. Each plate is 10 cm high (perpendicular to the field), 10 cm long (parallel to the field) and separated from the other plates by 5 cm. To support the four plates, a mounting structure runs along the length of the array. Having four plates, instead of one large one as been done previously [Stenzel and Gekelman, 1975a], allows a degree in flexibility in the way waves can be launched. For example, one can limit the k-spectrum which is launched by phasing the plates relative to one another [Fischer and Krämer, 1992]. Alternatively, one can phase all the plates together to launch a broad spectrum of lower hybrid waves. In preliminary experiments, it was found that the electric field signal in the striation wall was larger when the lower hybrid waves were launched in this broad-spectrum mode (it will be shown that the region of interest is the striation wall). Therefore, for all the experiments described in this paper, the four plates were kept at the same phase.

The launcher was moved into the plasma column to within 2 cm of the depletion boundary in order to ensure that the wave, whose energy moves very slowly across the field, interacts with the striation. A phase locked wave-packet of lower-hybrid frequency radiation is applied to the plates to launch the wave. The waveform applied to the exciter plates is identical from shot to shot.

Since the wave launcher does not preferentially launch any value of k-vector, the launched pulse is fairly broadband in k-space.
Wave electric fields were measured with a small dipole probe which measures the fields in the two directions perpendicular to the background field ($\hat{x}$ and $\hat{y}$). See Figure 4 for a schematic of the experimental setup and the coordinate system. The dipole probe actually comprises two mutually orthogonal dipoles which allows one to measure the electric field in the two axes perpendicular to the background field at once. Each dipole probe is approximately 4 mm from tip to tip ($\approx 200 \lambda_{\text{Debye}}$). Each dipole probe is connected to a $180^\circ$ hybrid tee outside the vacuum chamber. The tee is a device that effectively subtracts one signal from the other while maintaining a current path between the two probe tips. The electric field is proportional to the current in the tee so the current path must be complete to accurately measure an electric field [Stenzel, 1975]. The electric field measurements presented are not absolutely calibrated. The anisotropic susceptibility of the plasma prevents dipole probes with a tip-to-tip separation larger than a Debye length from being calibrated [Stenzel, 1975]. Before its use in the plasma, the probe was bench-tested in a large parallel-plate capacitor to ensure that the two probes measure mutually orthogonal fields.

Density measurements (from a Langmuir probe) and electric field measurements (from the dipole probe) are made in planes perpendicular to the background field. See Figure 4. These planes are presented in the next section. Each of the planes is actually composed of measurements at between 400 and 1681 individual locations in the plasma. At each of these positions, a time series, representing the evolution of the electric field in time, is averaged over 10 shots and digitized. The averaging is employed to improve the signal to noise ratio. The probe moves to each location, the wave form is digitized, and the probe is repositioned.

**Data**

Three different striations were studied in these experiments. The first is a cylindrical
striation (circular cross-section) with a 2.5 cm diameter (6.4 gyro-orbits across); the second is a slab striation (rectangular cross-section) with a 2.5 cm width; the third is a cylindrical striation with a 1.0 cm diameter (2.56 gyro-orbits). In each of these cases the skin depth, \( \delta = \frac{c}{\omega_{pe}} \), of the background plasma is 0.75 cm. For each of these striations, data characterizing the plasma density will be presented. For striation one, plots of electric field vectors as a function of x and y and as a function time will be presented; for striations two and three plots will examine the three-dimensional characteristics of the interaction. Finally, some of the data will be reexamined with an eye toward phase variations within a data plane. In these cases electric field data was acquired in up to five parallel planes and at 1600 locations on a rectangular grid at 0.25 mm spacing at 40 positions in both x and y directions. Vector plots shown in this paper do not show the data at full resolution because they would be too cluttered.

STRIATION ONE (cylindrical striation D =6.4 \( r_c \))

Figure 5 shows the density around striation one. The top panel has a fill pattern and contours which display the density as a function of position; the middle panel shows the density gradient (\( \nabla n/n \)); the bottom panel is a line-cut across each at \( y = 0 \) cm. The asymmetry in the density toward the top is caused by the support for the paddle used to create the striation. The striation is very deep; the density in the bottom is only about 1/7 the density outside. The walls of the striation are very steep; the density gradient scale length is approximately equal to the ion gyro-orbit.

A phase-locked wave packet of lower hybrid frequency radiation was launched into this striation. Figure 6 shows the measured electric field around the striation at one time. Density contours are shown for reference and are the same as those from Figure 5. \( f_{\text{wave}} = 6 f_{\text{lh}} \). The electric field vectors point around the striation along lines of constant density. The field is largest in areas of largest density gradient. The asymmetry in the
density is clearly visible in the asymmetry of the electric field. The field pattern is not electrostatic; i.e. $\nabla \times E \neq 0$; $A \neq 0$; a closed-path integral of electric field is not zero. A similar plot at lower frequency ($f_{\text{wave}} = 2 f_{\text{th}}$) would be qualitatively identical: the field pattern is independent of frequency in the frequency range $f_{\text{th}} \leq f_{\text{wave}} \leq 7 f_{\text{th}}$.

Figure 6 plotted $E_{\perp}(x,y)$ at one time. Figure 7 shows six plots of $E_{\perp}(x,y)$ taken over one half wave period. One can see that the vectors start at one extremum (right-hand into the page) in frame 1; go through a null in frame 3; and grow to the other extremum (right-hand out of the page) in frame 6. The wave electric fields oscillate at the wave frequency rotating from into the page and back out again. This behavior, too, is independent of frequency.

The electric field data can be averaged over time to find the energy in the wave’s electric field. A plot of time-averaged energy density is shown in Figure 8. The top panel shows the energy density with contours and fill. The middle panel takes the contours of the top panel and the fill representing the density gradient from Figure 5. It becomes clear that the regions of largest density gradient correspond to the regions of largest electric field amplitude and energy density.

**STRIATION TWO (Slab Striation; D = 6.4 r_c)**

Plots of density and density gradient, similar to those made for Striation One (Figure 5), are presented in Figure 9. These data concentrate on one end of the slab striation. Once again, the electric fields point around the striation; are largest in areas of largest density gradient; are not electrostatic; reverse every half wave period; and are independent of frequency.

One can examine the evolution of the wave pattern for this striation in three spatial dimensions. Figure 10 shows the measured electric field ($f_{\text{wave}} = 2 f_{\text{th}}$) in five
planes down the z-axis of the machine, along the background magnetic field. Though the five planes in Figure 10 are not spaced evenly along a wavelength, and do not appear as evenly spaced as the planes in Figure 7, the phase shifts between planes are clearly visible. By examining these phase shifts, the parallel k-number (or, equivalently, the parallel wavelength) can be found. We next take a Fourier transform of the wave electric field data in the perpendicular planes at a fixed time to find a wavenumber spectrum. It is obvious from the electric field patterns in figures six and seven that the dominant mode in the perpendicular direction is a cavity mode and not freely propagating waves. The amplitude and phase of the Fourier components are calculated for 1024 time steps. The wavenumber spectra are then averaged over time. The two dimensional Fourier transform assumes that the observed interference pattern is due to a large number of plane waves with differing wavenumbers, amplitudes and phases. The assumption of plane waves cannot be justified a priori because the plasma parameters are not uniform. However, it is instructive to investigate the wavelengths present under the plane wave approximation.

The results of the $k_\perp$ wavenumber spectra are shown in the lower panel of figure 11. Figure 11 shows the results of such a comparison. In the top panel, dispersion curves for the densities inside and outside the striation are plotted; these two curves are meant to be bounding cases for the plasma in the experiment. The top panel also shows a plot of the measured $k_\parallel$ spectrum created when the distribution of measured parallel k-numbers was fit as a gaussian distribution. Note that the peak in the $k_\perp$ spectrum corresponds to a wavelength slightly larger than the characteristic size of the striation. If points representative of the measured $k_\perp$ spectrum are projected to the upper panel one sees that the corresponding values of $k_\parallel$ fall on the linear dispersion curve. To illustrate this a single dotted line is projected upwards at the maximum of the $k_\perp$ spectrum. There is very good agreement between the theoretical and measured values, not only for the case shown, but also for the interaction at a higher frequency.
Figure 12 shows the density and gradient for Striation Three. This is the narrowest striation, dia = 1 cm, it is approximately one electron skin depth in diameter ($\delta$ is 0.75 cm outside the striation and 1.19 cm inside it). The asymmetry from the paddle support points up to the right. Again, the electric fields point around the striation; are largest in areas of largest density gradient; are not electrostatic; reverse every half wave period; and are independent of frequency. As with the data from Striation Two, one can find the phase shift among five planes arrayed down the machine to find $k_\parallel$. Figure 13 is a plot of measured $k$-values for Striation Three for the interaction at $f_{\text{wave}} = 2 f_{\text{th}}$. The peak in the $k_\perp$ spectrum is slightly larger than the diameter of the striation. The perpendicular wavelength derived from the analysis described above is $\lambda_\perp = \frac{k_\perp}{2\pi} \approx 1.57$ cm. However this is misleading since this is clearly not a plane wave. Once again the perpendicular wavenumbers lie on the cold homogeneous dispersion relation.

**PHASE EXAMINATION**

We now return to the data first presented in Figure 6, the electric field around Striation One (D = 6.4 $R_\parallel$). All of the phase analysis which follows is for this striation. We can present these same data in an entirely new way. First, we can find the center of the wave pattern and take the $E_x$ and $E_y$ data and convert them into $E_r$ and $E_t$, the radial and tangential (azimuthal) components of the electric field. Finding the center of data which are not truly circular was done by locating the position which minimized the radial component of the electric field. Once one has computed $E_r$ and $E_t$ at each point, one can take the time series and perform a Fast-Fourier Transform. Selecting out the wave frequency from the frequency spectrum, one can find the amplitude of each component at each position; the results are plotted in Figure 14. The top panel shows the large azimuthal field, the lower panel, the radial field. The circle and line shown in this plot
will be referred to in later figures. As one would expect from Figures 6 and 7, the field is primarily azimuthal.

Using the same FFT from which we found the amplitude, we can also calculate the phase in the complex plane at each position. For example for $E_r$, $\Phi_r = \tan^{-1}\left(\frac{\text{Im}(E_r)}{\text{Re}(E_r)}\right)$ where the real and imaginary parts come from the FFT of the time series. The variations of this phase in space are inversely proportional to the phase velocity of the wave. In other words, if one plots the phase as a function of position, the phase velocity of the wave is inversely proportional to the slope of that curve.

For the time being we set aside the radial field and concentrate on the azimuthal field since the fields in the striation are almost entirely tangential. Figure 15 shows the measured phase of the tangential field at every point in the measurement plane. The circle and line in Figure 15 are the same as in Figure 14. The phase along each of these paths is shown in Figure 16. For the azimuthal path, $0^\circ$ is along the x-axis and the angle increases moving counter clockwise. For the radial path, $0$ cm is at the lower left-hand corner of the data.

As one can see in the top panel of Figure 16, the phase does not vary much azimuthally. This is to be expected. Looking at the data in Figures 6 and 7, one does not see azimuthal variations in the phase of the wave. The phase does vary significantly along the radial cut, however. In other words, there is a radial phase dependence of the azimuthal (larger) electric fields. One can plot the phase variation with distance. The slope of this curve ($d\phi/dr$) is the locally measured wavenumber. Since the phase velocity is inversely proportional to the wavenumber one can find the locally-measured phase velocity by examining the slope of the variation of phase with distance. For these data, the slope one measures is largely independent of which radial path one chooses. In the first half of the encounter, the measured phase velocity along the path shown is $v_{ph} = 2.9$
× 10^8 cm/s; in the second half, \( v_{ph} = -1.9 \times 10^8 \) cm/s. Note that these values only take into account the radial dependence of the dominant (azimuthal) field.

It is important to ask whether it is reasonable to find phase shifts in the data plane. One likely explanation of the phase shifts is as follows: the density dependence of the waves warps the axial phase fronts. In a homogeneous plasma, the phase fronts whose k-vector is along the background field are planar. Since the plasma is non-uniform, one expects these phase fronts to warp. A phase shift measurement in a plane perpendicular to the background magnetic field then will measure shifts proportional to the effect of the plasma inhomogeneity.

While we can find the phase at any point, it is important to remember that the accuracy of the phase measurement is dependent on the amplitude of the electric field at that point. Small electric fields, whether azimuthal and radial, or cartesian, lead to relatively large uncertainties in phase. The “spike” in phase in Figure 15 (above and to the right of the center, near 140°) occurs where the amplitudes of the tangential field are smallest. Ideally one would like to have even more closely-spaced measurements, in three dimensions. In this experiment the spacing taken on planes perpendicular to the background magnetic field was 2.5 mm while the five planes at different locations along the device axis were 31.5 cm apart. Future studies searching for fine scale parallel structure necessitate higher-resolution measurement along \( B_0 \).

By traversing the striation over several different chords we have found that the direction of the measured phase velocity is heavily dependent on the path taken. Along some paths, these phase shifts appear as phase velocities which are constant across the striation; along other paths, the phase velocity appears to changes sign across the striation. In Pinçon et al [1997] and Bonnell et al [1998] it is shown that the electric field phase velocity reverses across the striation. In Schuck [1998], the phase velocity direction
remains constant. In other words, the laboratory data are consistent with the ionospheric observations to completely understand the phase shifts, it may be necessary to plot the three-dimensional phase fronts.

**Summary of Results & Conclusions**

The experimental results can be summarized as follows: wave electric fields point around the striations, along lines of constant density. The fields are greatest in the regions of largest density gradient. The pattern resembles that of an azimuthal mode with \( m=0 \). These results are independent of frequency and striation size and shape throughout the ranges explored in these experiments. The field pattern is not electrostatic. The perpendicular wavenumber is set by the characteristic size of the striation. The parallel wavenumber spectrum is described very well by the cold homogeneous dispersion relation. The dispersion relation’s determination of the parallel wavenumber implies that the depletion must have a minimum length along the background field in order to support a non-evanescent mode. In the laboratory the minimum wavelength ranged between 30 cm and 115 cm; for space data, that length is on the order of 500 m. Wavenumber spectra of the electric field structure were also obtained in perpendicular planes. The spectra reflect the structure of the cavity and also have peaks which lie on the linear dispersion curve. This is an interesting observation but can only be quantified by comparing the results with an electromagnetic theory in the presence of a density cavity.

Comparisons between laboratory data and ionospheric observations are often difficult. However, the laboratory results agree well with some published observations. Compare, for example, Figure 8 in this paper with Figure 1, panels E-F and I-J in McAdams [1998] and Figure 2 in Delory [1997]. In each case, it is clear that the wave electric fields are largest in areas of largest density gradient.
The theory developed by Schuck et al. [1998] to explain these rocket observations is electrostatic and does not consider m=0 modes. In papers comparing the theory to observations, [Schuck, 1998; Bonnell, 1998; Pinçon 1997], the authors find good agreement. Even so, a brief examination of Figure 6 makes it clear that the laboratory work can not be described by an electrostatic theory. It is important to remember that though lower hybrid waves’ dispersion can be well-described by an electrostatic theory, wave magnetic fields play an important role in the physics of these waves. The fact that the laboratory experiments are scaled to recreate ionospheric physics implies that our results should be comparable to ionospheric observations. Certainly the measured k-spectra for the laboratory experiments show that the laboratory data satisfy any reasonable requirements about what constitutes an electrostatic wave; see for example Figure 13 for Striation Three where the striation diameter is close to the collisionless skin depth. The fact remains that the laboratory data differ greatly from the picture of an electrostatic wave.

There are two possible reasons for this effect. The first is that there is some key parameter of which we are not aware. We have scaled our experiment to meet, as closely as possible, all the key parameters from the ionosphere. If there is some other as-yet-unknown parameter, it may be that we are measuring a phenomenon different from that measured in the ionosphere. For example, if the ionospheric striations are all shorter than one half parallel wavelength, we would be measuring a different mode. More ionospheric measurements, preferably multi-point measurements, may also hold the key to understanding this possibility. And since there is such a wide range of ionospheric conditions, it seems likely that if the laboratory data differ from some published rocket observations, there are other rocket encounters which more closely resemble the laboratory data.

The other possibility is that we are measuring in the laboratory the same mode measured in space. If this is the case then it is clear that a full electromagnetic theory is
necessary to understand the physics of the phenomenon. In addition, future measurements
could be made with an eye toward measuring a completely different field structure than
has been considered in the past. Furthermore, reexamination of existing observational
data may yield examples of interactions which more closely resemble the laboratory data.

A recent analysis of FREJA data [Kjus et al, 1998] discusses the basic statistical
characteristics of lower hybrid waves and associated density depletions. In the observational
part of their paper they state that in “approximately 10% of all the cases” the wave
energy has a double humped appearance as the satellite flew through the striation. Based
on the data they accumulated they speculate that “most of the cavities are actually associated
by a ring distribution of wave energy and the observations without any bifurcation are
merely a consequence of the satellite glancing the cavity along a trajectory, avoiding the
central wave energy depletion”. In our experiments the wave energy is localized on the
density gradient and a fly through the cavity would lead to a double hump in wave energy
as well. All spacecraft and rocket data have the same problem. They cannot map a
perturbation in two (or three) dimensions and do not know if they intersect a density
striation along a chord or have gone through it’s center. This could be resolved by future,
multiple spacecraft missions. The power of a controlled laboratory experiment is to
remove such doubts by making fully three dimensional measurements.

In conclusion, we have shown that when lower hybrid wave radiation impinges on
a density striation in a controlled experiment, the interaction produces large field amplitudes
in the gradient regions. This is the most important result of these experiments. The
increased fields occur over a wide range of conditions, indicating that this phenomenon
may be widespread in space. Furthermore, the increased amplitude occur in a small area
in space. Some have predicted that such conditions may be fertile ground for ion
acceleration [Reitzel, 1996]. Future experiments should investigate this possibility. In
addition, future experiments could provide a more complete comparison with ionospheric observations by launching lower hybrid waves with a modulated current beam.

The electric field pattern is electromagnetic in nature and is complicated by phase shifts in two-dimensions that are not easily characterized. The three-dimensional spectrum, however, is described well by a simple dispersion relation. The generally good agreement between laboratory data and ionospheric data suggest very strongly that experiments performed in the laboratory have the ability to aid the interpretation and understanding of ionospheric phenomena. These results can provide a guide-post to help direct future theoretical efforts and further observations and gain a more complete insight into the lower hybrid wave—striation interaction.
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Captions

Figure 1  The lower hybrid frequency as a function of density for several magnetic fields. For these experiments, the background magnetic field is 1650 G, and the lower hybrid wave is most purely density dependent between $1.0 \times 10^{11}$ cm$^{-3}$ and $1.0 \times 10^{12}$ cm$^{-3}$.

Figure 2. Two curves showing $k_\parallel$ ($k_z$) vs $k_\perp$ ($k_x$) for these experiments. Note the different scales for $k_\parallel$ and $k_\perp$. The density is taken to be $5 \times 10^{11}$ cm$^{-3}$, the density of the plasma outside of the striation.

Figure 3. Schematic of the LAPD. The plasma is created when electrons are emitted by the cathode and fully ionize the background gas before being absorbed by the anode. Measurements are made in the plasma via the access ports.

Figure 4. Three dimensional schematic of the experimental setup. The wave-launching array is moved close to the striation boundary to ensure that the wave energy interacts with the striation. Measurements are made in x-y planes perpendicular to the background field.

Figure 5. Density and density gradient around Striation One (cylindrical striation with a diameter of 2.5 cm = 6.54 $r_i$). The top panel shows density as a yellow-purple scale pattern and with contours. The middle panel shows the density gradient. The bottom panel shows a cut across the top two panels at $y = 0.0$ cm. The asymmetry toward the top of the striation is caused by the support for the paddle which creates the striation. Grid lines are spaced at 1.0 cm. The striation is formed by a 2.5 cm diameter paddle centered near $(x,y=0)$, which blocks the ionizing electrons during plasma production.

Figure 6. Electric field around Striation One at one time for one frequency ($f = 2f_{ih}$). The contours show the density, and are taken from the top panel of Figure 5. The electric
fields point around the striation, along lines of constant density. Grid lines are spaced at 1.0 cm; this plot is zoomed in closer than Figure 5. The bottom panel shows a cut across the electric field at y = 0.0 cm. The asymmetry seen in density is visible in the azimuthal asymmetry of the electric field pattern. In this and all of the following vector plots, vectors which have a smaller magnitude than the error bars shown in this figure are not drawn as they are in the noise and have random orientations.

Figure 7. The time evolution of the electric field in striation 1. Time evolves down the left column then down the right column. In the first panel (upper left), the field is at a local maximum in time; the sense of rotation points into the page. The field progresses through a minimum in panels three and four. Finally the field reaches another maximum with the sense of rotation out of the page (180° out of phase). The wave frequency is 90 Mhz and the time interval between the first and last panel is 5.56 nsec. The digitization rate is 5 GHz.

Figure 8. Time-averaged energy density around striation one for a single frequency (f = 6 \( f_n \)). The top panel shows the energy density in gray-scale fill and with contours. The middle panel shows the density gradient from Figure 5 in fill; and the energy density contours from the top panel to allow an easy comparison. The bottom panel shows a line-cut across y = 0.0 cm. The paddle and striation diameters are 2.5 cm.

Figure 9. Density and density gradient around Striation Two (slab striation with a width of 2.5 cm =6.54 \( r_a \)). The top panel shows density as a gray-scale pattern and with contours. The middle panel shows the density gradient. The bottom panel shows a cut across the striation at y = 4.0 cm.

Figure 10. Electric field vectors in four axially aligned planes at one time. The planes are not evenly spaced. In the top left panel (z = 94.2 cm), the field points up on the left
hand side; at \( z = 440.2 \) cm, the field there points down. The panel at \( z = 220.1 \) cm shows a null. One can find the parallel wavelength using such data.

Figure 11. Measured perpendicular and parallel wavenumber spectra for the interaction with Striation Two. The top panel shows two dispersion curves for the wave at the launched frequency \( (f = 30 \text{ MHz} = 2 \, f_0) \). The dotted line represents the density outside the striation, the solid line the density at the bottom of the striation. The bottom panel shows the spectrum of measured \( k_\perp \) for the interaction of the wave with Striation Two. The peak corresponds to a wavelength somewhat larger than the striation width. Note that \( E_x \) and \( E_y \) have different spectra due to the asymmetry of the fields in \( x \) and \( y \). By projecting the peak in measured \( k_\perp \) up to intersect with the dispersion curve, one can find the expected \( k_\parallel \) spectrum. The measured \( k_\parallel \) spectrum is indicated by the gaussian distribution on the top panel.

Figure 12. Density and density gradient around Striation Three (cylindrical striation with a diameter of \( 1.0 \text{ cm} = 2.56 \, r_0 \)). As in Figures 5 and 9, the top panel shows density; the middle panel shows density gradient; the lowest panel is a line cut at \( y = 0.0 \) cm.

Figure 13. Measured perpendicular and parallel wavenumber spectra for the interaction with Striation Three. The top panel shows two dispersion curves for the wave at the launched frequency \( (f = 30 \text{ MHz} = 2 \, f_0) \). The dotted line represents the density outside the striation, the solid line the density at the bottom of the striation. The bottom panel shows the spectrum of measured \( k_\perp \) for the interaction. The peak corresponds to a wavelength somewhat larger than the striation diameter, for this case at a significantly larger value of \( k_\perp \). By projecting the peak in measured \( k_\perp \) up to intersect with the dispersion curve, one can find the expected \( k_\parallel \) spectrum. The measured \( k_\parallel \) spectrum is indicated by the gaussian distribution on the top panel.
Figure 14. Azimuthal and radial wave amplitudes for striation 1 (D= 6.4 R_\text{ci}). The wave data were transformed into polar coordinates and the amplitude was found at each point. Both panels use the same scale (in arbitrary units). The circle and line will be referred to in upcoming figures and are provided here for reference. The circle is a path at constant radius; the line is a diameter cut.

Figure 15. Measured phase of the azimuthal wave field for striation 1. The circular path is at a constant radius; the line is a radius cut. Refer to Figure 14 to see how these paths intersect the amplitude.

Figure 16. The measured phase along the two paths through the plane of phase for striation 1 (D= 6.4 R_\text{ci}). The top panel shows the measured azimuthal phase from 0° to 360°, starting on the x-axis (the right-facing horizontal) and moving counter-clockwise. There are only small variations in phase around the striation. The bottom panel shows the measured radial phase starting in the lower left, and moving along the 4.0 cm long path. Phase changes significantly along this, and any other radial path. The uncertainty in phase largest in areas of smallest wave amplitude.
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Table 1. Comparison of key parameters for these experiments. $\sim$ is the wave frequency; $f_{lh}$, $f_{ce}$, $f_{pe}$ are the lower hybrid, electron cyclotron and electron plasma frequencies respectively; D is the striation diameter; $r_{ci}$ is the ion Larmour radius; L is the density gradient scale length; $\lambda_{\perp}$ and $\lambda_{||}$ are the perpendicular and parallel wavelengths and $\delta$ the electron skin depth.