Nonlinear Landau damping of resonantly excited fields in nonuniform plasmas

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In this study the resonant excitation of a nonuniform plasma by two electromagnetic waves at closely spaced frequencies $\omega_1$, $\omega_2$ with $\omega_1 > \omega_2$ is considered. Each of the pump wave excites a Langmuir wave at the point in the density profile, where plasma resonance is achieved. For profiles having scale lengths $L$ large compared to the characteristic Airy scale length (i.e., $k_A L \gg 1$) the linear Landau damping of the Langmuir waves is quite small close to their cutoff, hence the beat interaction with slow electrons plays an important role. The problem is formulated in terms of differential equations in configuration space for the waves at $\omega_1$, $\omega_2$, which are coupled to the idler field at $\omega_1 - \omega_2$. The idler is described kinetically by a differential equation in Fourier space. The interaction is governed by the parameter $\delta z = (1 - \frac{\omega_2}{\omega_1}) L$, and the scaled pump strength. For $\delta z < 1$ the ponderomotive nonlinearity is recovered and a dissipative contribution is obtained. For $\delta z > 1$ plasmaon transfer from $\omega_2$ to $\omega_1$ causes strong depletion of the high-frequency wave before significant ponderomotive profile changes set in. The fractional power absorbed through the idler is smaller than the power transferred to $\omega_2$ by a factor of $(1 - \frac{\omega_2}{\omega_1})$.

I. INTRODUCTION

In uniform plasmas, nonlinear Landau damping (a process more accurately named beat damping) is known to play a key role in transferring energy and momentum of large phase velocity waves to bulk plasma particles. The underlying concept consists of the beat excitation of an idler (not a collective mode) electric field having frequency $\omega_1 - \omega_2$ and wave number $k_1 - k_2$, arising from the bilinear interaction of collective modes having frequencies and wave numbers $(\omega_1, k_1)$ and $(\omega_2, k_2)$, respectively. In addition to the generation of a strong wave-particle interaction for those particles whose velocity $v$ satisfies the beat resonance condition $v = (\omega_1 - \omega_2)/(k_1 - k_2)$, the idler field allows a transfer of wave action (or quanta) from the higher-frequency mode, say $\omega_1$, to the lower frequency one, $\omega_2$. It is this cascading aspect of the process that is sought to contribute to the spectral broadening of narrow spectrum instabilities and launched coherent signals within the framework of weak turbulence theory.

Although nonlinear Landau damping has been extensively investigated, both theoretically and experimentally, for various plasma collective modes, the majority of the studies have concentrated on its properties in uniform plasmas. Most applications of this process to nonuniform plasmas have consisted of Wentzel, Kramers, and Brillouin (WKB) generalizations of the uniform medium results. By contrast, it is the purpose of the present study to illustrate a physical situation in which the nonuniformity of the plasma plays a key role, namely the resonant excitation of a nonuniform plasma. Aside from its intrinsic basic interest, this problem has relevance to laser-plasma experiments, ionospheric heating by high-frequency (HF) waves, and electron acceleration in the auroral ionosphere.

Specifically, the present study considers a warm magnetized plasma whose zeroth-order density $n_0$ varies linearly with position $z$, as illustrated in Fig. 1. We envision the plasma to be externally excited by two coherent electromagnetic waves at closely spaced frequencies $\omega_1$ and $\omega_2$ with $\omega_1 > \omega_2$, and $(\omega_1 - \omega_2)/\omega_1 < 1$. Near plasma resonance, the density profile is taken to be a linear function of position, with $1 - \omega_2^2(z)/\omega_1^2 = z/L$, where $\omega_p$ is the local plasma frequency and $L$ is the density scale length. As can be shown rigorously, the mode conversion process that occurs at plasma resonance can be described by replacing the electromagnetic fields by uniform (capacitor-like) pumps of the form $E_0 j \exp(-i\omega_j t)$, $j = 1, 2$, and $i$ is the time. The complex amplitudes $E_{0j}$ can be related to the incident power of the external electromagnetic signals.

The linear response of the warm plasma consists of the resonant excitation of two Langmuir waves at frequencies $\omega_1$ and $\omega_2$, whose amplitudes are sketched in Fig. 1 by the solid curves whose peaks appear close to the points $\omega_j = \omega_p$. Since we choose the origin ($z=0$) to coincide with the cutoff for the Langmuir wave at frequency $\omega_1$, the wave pattern of the second wave at the lower frequency, $\omega_2$, is shifted by a distance $z_2 = L[1 - (\omega_2/\omega_1)^2]$. To obtain a better appreciation of the relevant parameter dependences in the problem, we note that each of the driven-Airy patterns exhibits a maximum phase velocity that scales as $(v_A) = \omega_0 / k_A$, where the characteristic Airy-wave number is $k_A = (k_D L/\sqrt{3})^{2/3} L^{-1}$, and the Debye wave number is defined as $k_D = \omega_0 / \bar{E}$, with $\bar{E}$ the electron thermal velocity defined as $\bar{E} = \sqrt{T_e/m}$. This implies that the effective phase velocity of the individual electric fields driven near the local plasma resonances scale as $(v_A) / \bar{E} = (\omega_0 / \omega_1)(3k_D L)^{1/3}$. Consequently, for plasmas having large values of $(k_D L)^{1/3} (> 5)$, as may be encountered in laboratory and space applications, the resonantly driven Langmuir waves experience relatively small Landau...
damping, unless a fast electron tail population is present. Of course, since the WKB phase velocity of Langmuir waves decreases as they propagate in the direction of decreasing density, eventually the waves experience strong Landau damping, provided nonlinear processes do not play a role at distances close to their generation points, i.e., \(z = 0\) and \(z = z_2\). One such nonlinear process is nonlinear Landau damping.

Figure 2 illustrates the spatial dependence of the WKB phase velocity of each of the resonantly excited Langmuir waves, i.e., \(\omega_j / k_j(z)\), with \(k_j(z) = k_D[(z - z_j)/3L]^{1/2}\), and \(z_j\) is the respective cutoff position. The phase velocities are scaled to \(\nu_\parallel\) and the relevant scaled spatial coordinate is \(\xi = k_A x\). Figure 2 also shows the spatial dependence of the WKB group velocity, i.e., \(u_j(z) = 3(k_j/k_D)\). The latter quantity being of relevance in describing the beat resonance velocity \((\omega_1 - \omega_2)/[k_1(z) - k_2(z)]\) for closely separated frequencies. The WKB picture that emerges from Fig. 2 is that each of the resonantly excited waves undergoes negligibly small Landau damping for \(\xi < \left(\frac{k_D}{2}\right)\), while the beat resonance sweeps across the bulk of the electron distribution function. The consequence of this behavior is that at sufficiently large pump amplitudes, significant beat coupling results, and is accompanied by strong depletion of the wave having higher frequency \(\omega_1\). Simultaneously, the amplitude of the lower frequency wave, \(\omega_2\), must increase in order to conserve the plasmon number. Of course, beyond this stage the second wave can trigger other strong nonlinear processes such as particle trapping and phase independent acceleration of both background electrons and particles, whose initial velocity is boosted by beat resonance with the idler field.

Although the results of the present study indeed corroborate the general qualitative features expected from the WKB picture suggested in Fig. 2, several detailed features are determined by the non-WKB properties of the driven-Airy pattern, which are most pronounced where the field amplitude is largest, i.e., within the first lobe of the Airy pattern. In fact, because strong nonlinear Landau damping occurs within the first lobe, it is relatively difficult to realize a situation in which two propagating waves interact in the manner described for uniform plasmas.

While the WKB picture shown in Fig. 2 is not useful in making a direct quantitative prediction of the role of nonlinear Landau damping in the resonant absorption process, it is quite valuable in suggesting what approximations can be made in the correct mathematical description of the process. First of all, for \((k_D L)^{1/3} < 1\), linear Landau damping of the primary waves at \(\omega_1\) and \(\omega_2\) is negligible in the region where the beat resonance sweeps across the electron velocity distribution function, so that the collective response at \(\omega_1\) and \(\omega_2\) can be accurately represented by a warm-fluid description. This implies that the spatial evolution of the electric fields at \(\omega_1\) and \(\omega_2\) is determined by a differential equation in configuration space, in which pumping by both the external source and beat charge oscillations must be included, but kinetic linear damping effects can be ignored. However, the response of the plasma to the idler field does not admit a fluid description since the beat-resonance velocity is smaller than the thermal velocity of electrons. Consequently, the response at the low-frequency \(\omega_3 = \omega_1 - \omega_2\) must be described kinetically, and since one is dealing with a spatially nonuniform plasma, this requires a differential equation in Fourier transform space. The full problem then consists of three coupled equations: one in Fourier space and two in configuration space.

The system of scaled equations [Eqs. (19)-(21)] that govern this problem depend explicitly on three scaled parameters: the frequency separation \(\omega_3 = 1 - \omega_2 - \omega_1\), the pump strength \(p_j = (k_D L E_{0j})^2/24\pi n_0 T_e\), and the gradient scale length \(k_A L\). However, for large scale lengths...
(as assumed in this study) the explicit dependence on $k_A L$ does not play an important role. In the limit $\omega_3 = 0$, the system of equations reduces to a single equation (depending on a single pump parameter $p$) introduced by Morales and Lee to describe the role of ponderomotive density changes on resonant absorption. Consequently, the present study provides a generalization of this process to nonzero frequencies. For $\omega_3 \neq 0$ the ponderomotive effect is no longer adiabatic and energy absorption by slow electrons results. The efficiency of this absorption is found to be proportional to $\omega_3$. However, this is not the main process that causes the nonlinear damping of the higher-frequency wave. It is plasmon transfer from $\omega_1$ to $\omega_2$ that dominates the interaction. For $\omega_3 > 1$ the higher-frequency Langmuir wave predominantly interacts with the first lobe (where $k$ is nearly zero) of the lower-frequency wave. In passing through this lobe, the $\omega_1$ wave is strongly damped for values of $p_j < 0.4$, which is slightly below the required pump amplitude to cause significant ponderomotive modifications.

The paper is organized as follows. In Sec. II the mathematical formulation is presented. In Sec. III analytic models pertaining to the problem are developed. The conservation laws for the system are discussed in Sec. IV. Numerical results are described in Sec. V and conclusions are presented in Sec. VI.

II. MATHEMATICAL FORMULATION

The geometry of the problem being considered is sketched in Fig. 1. A plasma with a linear density gradient of scale length $L$, i.e., $n_L = n_0(1-z/L)$ is irradiated by two electromagnetic waves at frequencies $\omega_1$, $\omega_2$, with $\omega_1 > \omega_2$. The location $z=0$ corresponds to the plasma resonance at $\omega_1$. In the neighborhood of the resonance the electromagnetic waves are approximated as uniform (capacitor-like) pumps. The self-consistent spatial structure of the primary electric fields at frequencies $\omega_1$ and $\omega_2$ are determined by Poisson’s equation in $k$ space

$$E_3(z) = \mathcal{F}^{-1}[\tilde{E}_3(k)] = \frac{1}{2\pi} \int_{-\infty}^{\infty} dk \exp(+ikz) \tilde{E}_3(k),$$

and similarly for the complex amplitudes of the distribution functions. The spatial Fourier transform is applied to Eqs. (2) and (3) to solve for the unknown complex amplitudes of the distribution functions $f_{3j}(k)$. From these we determine the charge density that results in the following Poisson equation in $k$ space

$$\frac{i}{L} \left[ 1 - \epsilon(\omega_3,k) \right] \frac{d \tilde{E}_3(k)}{dk} + \epsilon(\omega_3,k) \tilde{E}_3(k) = \tilde{S}_3(k),$$

where $\epsilon(\omega_3,k)$ is the exact kinetic dielectric coefficient including ion and electron contributions and $\tilde{S}_3(k)$ is the Fourier transform of the bilinear source term at frequency $\omega_3 = \omega_1 - \omega_2$.

In Eq. (5), the derivative in $k$ space arises from the spatial nonuniformity of the zeroth-order density. In obtaining Eq. (6), it has been assumed, as is appropriate to this problem, that the contributions arising from $f_{1e} f_{2e}$ are well approximated by the cold plasma limit (i.e., large phase velocity).

In our model, the ratio of the plasma electron thermal velocity $\overline{v}$ to the typical phase velocity $v_A$ of the primary waves ($\omega_1$ and $\omega_2$) is considered to be small. Hence, for these waves ($j = 1, 2$), Eq. (1) can be well approximated in the lowest order by

$$-i\omega_3 f_{3e} + \frac{e}{m} \frac{\partial f_{3e}}{\partial v} + \frac{1}{m} E_3(z) \frac{\partial f_{3e}}{\partial v} = \frac{e}{m} \left( E_1(z) \frac{\partial f_{1e}}{\partial v} + E_2(z) \frac{\partial f_{2e}}{\partial v} \right),$$

$$-i\omega_3 f_{3j} + \frac{e}{m} \frac{\partial f_{3j}}{\partial v} + \frac{1}{M} E_3(z) \frac{\partial f_{3j}}{\partial v} = 0.$$
\[ \varepsilon_{\omega}(z, \omega_j) E_j(z) = E_{0j} + \frac{4\pi}{i\omega_j} J_j^{NL}(z), \] (7)

where \( \varepsilon_{\omega} \) is the warm fluid dielectric operator (in configuration space),

\[ e_{\omega}(z, \omega_j) = \frac{3}{k_{D} v} \frac{\partial^2}{\partial z^2} + \frac{z - z_j}{L} + i\Gamma_j. \] (8)

The normalized collision frequency for electrons is \( \Gamma_j = \nu_j \) and \( z_j \) is the position of the resonance layer at frequency \( \omega_j \). For \( \nu_j \neq v_{A} \), the linear Landau damping contribution to the structure of the primary wave fields is negligible near plasma resonance, the region of interest to this study.

The nonlinear current \( J_j^{NL}(z) \) in Eq. (7) is driven by the beat of the idler electric field \( E \) and one of the primary electric fields, i.e.,

\[ J_j^{NL}(z) = -e \left[ (n_2(z) v_2(z) + n_3(z) v_2(z)) \right], \] (9)

\[ J_j^{NL}(z) = -e \left[ (n_1(z) v_2(z) + n_3(z) v_1(z)) \right], \] (10)

where the subscripts 1, 2, and 3 represent the quantities at frequencies \( \omega_1 \), \( \omega_2 \), and \( \omega_3 = \omega_1 - \omega_2 \), respectively. The cold plasma approximation can be used to obtain the perturbed velocities and densities at high frequencies,

\[ v_j(z) = \frac{e E_j(z)}{m \omega_j}, \] (11)

\[ n_j(z) = \frac{-e m_0}{m \omega_j} \frac{\partial}{\partial z} \left[ \left( 1 - \frac{z - z_j}{L} \right) E_j(z) \right] (j = 1, 2). \] (12)

To obtain the perturbed density \( n_3(z) \) and velocity \( v_3(z) \) at \( \omega_3 \), a kinetic description is required. The procedure then consists of determining \( n_3(k) \) and \( v_3(k) \) using Eq. (2) and then transforming to configuration space,

\[ n_3(k) = \frac{e m_0}{2 i m k_0} \frac{\omega_3}{\sqrt{2 k_0}} \left( 1 - \frac{i}{L \frac{d}{dk}} \right) E_3(k) \]
\[ + \frac{e m}{\omega_1} \left[ \frac{E_1(z)}{\omega_1} \right] \frac{\partial}{\partial z} \left[ \left( 1 - \frac{z}{L} \right) E_1(z) \right], \] (13)

\[ Z(s) = \frac{1}{\sqrt{\pi}} \int_{-\infty}^{\infty} dt \exp(-t^2), \] (14)

\[ v_3(k) = \frac{\omega_3 n_3(k)}{k_0}, \] (15)

\[ n_3(z) = \mathcal{F}^{-1} [\tilde{n}_3(k)], \] (16)

\[ v_3(z) = \mathcal{F}^{-1} [\tilde{v}_3(k)]. \] (17)

It is useful to incorporate the natural Airy scaling associated with a nonuniform plasma, as follows:

\[ \kappa = k/k_A, \]

\[ \tilde{n}_3 = k_A L \frac{\omega_3}{\omega_1}, \]

\[ \tilde{v}_3 = k_A L \frac{\omega_3}{\omega_1}, \]

\[ \gamma_j = k_A L \Gamma_j; \quad j = 1, 2, \] (18)

\[ \rho_j = \frac{(k_{D} E_0)^2}{2 \pi n_0 T_e}; \quad j = 1, 2, \]

\[ E_j = k_A L E_0 \gamma_j; \quad j = 1, 2, 3, \]

\[ E_{03} = \left( \frac{e L}{T_e} \right) E_0 E_{02}, \]

where \( T_e \) is the electron temperature.

The corresponding system of scaled equations that describe nonlinear Landau damping of resonantly excited fields in a nonuniform plasma consists of three coupled equations for the electric fields. The high-frequency fields are described by differential equations in configuration space, whereas the idler field is obtained from a differential equation in \( k \) space,

\[ \left( \frac{\partial^2}{\partial \xi^2} + \xi + i \gamma_1 \right) \hat{S}_1(\xi) - \rho_2 S_1(\xi), \]

\[ \left( \frac{\partial^2}{\partial \xi^2} + (\xi - \xi_2) + i \gamma_2 \right) \hat{S}_2(\xi) = 1 + \rho_1 S_2(\xi), \] (19)

\[ i \frac{k_A L}{[1 - e(\tilde{\omega}_3, \kappa)]} \frac{d}{dk} \frac{\hat{S}_3(k)}{k} + e(\tilde{\omega}_3, \kappa) \hat{S}_3(k) = \hat{S}_3(k). \] (20)

The quantities appearing in the system of Eqs. (19)–(21) are

\[ \xi_2 = k_A L (1 - \tilde{\omega}_2^2), \] (22)

\[ \varepsilon(\tilde{\omega}_3, \kappa) = 1 - \frac{3 k_A L}{2} \kappa \left[ \frac{-3 \tilde{\omega}_3}{\sqrt{2 k_A L} \kappa} \right] + \frac{T_e}{T_i} \left[ \frac{3 \tilde{\omega}_3 \tilde{v}_3}{(2 k_A L) \kappa} \right], \] (23)

\[ S_1(\xi) = \frac{1}{\omega_2} \frac{\partial}{\partial \xi} \left[ \left( 1 - \frac{(\xi - \xi_2)}{k_A L} \right) \hat{S}_2 \right] \hat{S}_1^{-1} \left( \frac{i}{k_A L} \right) \] (24)

\[ S_2(\xi) = \frac{\partial}{\partial \xi} \left[ \left( 1 - \frac{\xi}{k_A L} \right) \hat{S}_1 \right] \hat{S}_2^{-1} \left( \frac{i}{k_A L} \hat{S}_1 \right)^\dagger \] (25)
where $\mathcal{F}_s$ and $\mathcal{F}_s^{-1}$ are Fourier transform and inverse Fourier transform operators defined in the dimensionless conjugate space $(\xi, \kappa)$, $T_i$ is the ion temperature, and $\bar{u}_i$ is the corresponding ion thermal velocity. Equations (19)-(21) are solved numerically, and the results are presented in Sec. V. However, we first discuss the results expected in two limiting cases that can be treated analytically, and derive conservation laws that can be used as checks of numerical accuracy.

### III. ANALYTICAL MODEL

#### A. Relation to ponderomotive force

As the frequency difference between the primary waves is decreased, i.e., $\omega_3/\omega_1 \to 0$, the two Langmuir waves excited at plasma resonance become indistinguishable within the coherence length of the wave packet. Then, the waves can be treated as a single high-frequency field and the beat interaction can alternatively be viewed as a ponderomotive force problem. Here we show that the kinetic beat-wave interaction model reduces to the well-known ponderomotive limit as $\omega_3 \to 0$, and obtain the generalization of the ponderomotive source term (including dissipation) at non-zero frequency.

We consider the limit $L \to \infty$ to compare the result with the uniform plasma case. In the limit of $\omega_2 \to \omega_1$, Eq. (5) using Eq. (6) reduces to

$$\epsilon(\omega_1, k) \tilde{E}_s(k) = -\frac{\epsilon}{\mathcal{F}_s(k)} \left( \frac{\partial}{\partial \xi} |E_1|^2 \right).$$

The quasineutrality condition of low-frequency oscillations implies that the unity term in the dielectric coefficient $\epsilon$ in Eq. (23) is small compared to the ion and electron susceptibilities. Since both ions and electrons are adiabatic in the limit $\omega_3 \to 0$, the dielectric reduces to

$$\epsilon(\omega_3 = 0, k) = \frac{k_D^2}{\mathcal{F}_s(k)} \left( 1 + \frac{T_e}{T_i} \right).$$

Using Eqs. (28) and (29) and $n_e(z) = n_i(z)$ yields

$$E_s(z) = \left( -\frac{\epsilon}{\mathcal{F}_s(k)} \frac{1}{(1 + T_e/T_i)} \right) \frac{\partial}{\partial z} |E_1(z)|^2,$$

which are identical to the expressions obtained by the usual ponderomotive calculations.

Furthermore, the nonlinear source term $S_1(\xi)$ in the limit of $\omega_3 \to 0$ takes the form

$$S_1(\xi) \approx i\mathcal{F}_s^{-1}(\Phi) \tilde{E}_1$$

$$\approx 2\mathcal{F}_s(\xi) \int d\xi \left( 3k_A L \tilde{E}_3(\xi) \frac{\partial}{\partial \xi} |\tilde{E}_1|^2 \right),$$

and using the corresponding limit of the idler field given in Eq. (21), reduces Eq. (32) to

$$S_1(\xi) \approx \frac{2T_e}{T_e + T_i} |\tilde{E}_1(\xi)|^2.$$ (33)

Finally, inserting this result in Eq. (19) yields

$$\left( \frac{\partial^2}{\partial \xi^2} + \xi |\tilde{E}_1^1(\xi)|^2 \right) \tilde{E}_1^1(\xi) = 1,$$

where

$$p = \frac{(k_D L)^2 E_0^2}{24\pi n_0(T_e + T_i)}.$$ (35)

Equation (34) is the model equation first introduced by Morales and Lee\textsuperscript{12} to investigate the nonlinear modification of resonant absorption caused by profile changes due to the ponderomotive force. It should be noted that the system of equations (19)-(21) constitutes the nonzero frequency generalization of this interaction; it requires a kinetic description associated with wave-particle resonances mediated by the idler field, i.e., nonlinear Landau damping.

#### B. Limit of well-separated peaks

As the frequency difference $\omega_3$ is increased, the location of the plasma resonance for the wave at frequency $\omega_2$ moves in the direction of decreasing density according to Eq. (22), i.e., in scaled coordinates $\xi_2 \approx 2\tilde{\omega}_3$. This implies that for sufficiently large $\tilde{\omega}_3$, the beat coupling is dominated by the interaction of a traveling Langmuir wave at $\omega_1$ with the resonant peak (Airy peak) of the electric field at $\omega_2$, as sketched in Fig. 3.

Next, we present a simple calculation of the nonlinear damping expected in this limit. The value of $\tilde{\omega}_3$ that naturally separates the small values of $\tilde{\omega}_3$ from large values is given by the condition that the two driven-Airy principal lobes be resolved in space. This occurs when the first peak of the pattern at $\omega_2$ falls on the second peak of $|\tilde{E}_1|^2$, i.e., $\xi_2 \approx 3.6$. This yields a characteristic frequency separating the small and large $\tilde{\omega}_3$ regimes,

$$\Delta \omega_{crit} \approx 1.8.$$ (36)

For $\tilde{\omega}_3 < \Delta \omega_{crit}$, the beat interaction becomes adiabatic, and results in the ponderomotive density modification previously described, except that for nonzero $\tilde{\omega}_3$ there is always a resistive contribution. To obtain an analytic estimate of
FIG. 3. The amplitudes of the unperturbed electric fields are shown for large frequency separation ($\omega_s > \Delta \omega_{\text{pl}}$). The beat interaction is mostly confined to the region of overlap of the two primary waves, shown shaded in the figure. The model presented in Sec. III approximates the field at $\omega_0$ as a constant over a region and treats the field at $\omega_1$ as a WKB traveling wave.

The nonlinear damping in the large $\omega_s$ limit we adopt a simple model in which $\mathcal{E}_2(\xi)$ is localized to the region of the first Airy peak at $\xi \approx \xi_2$, and the electric field $\mathcal{E}_1(\xi)$ is approximated by its WKB (asymptotic) form consisting of a traveling wave with scaled wave number $\kappa_1(\xi) \approx \sqrt{2} \omega_s$.

The scaled effective phase velocity of the idler field is $\omega_0/\kappa_1(\xi)$ because in the first lobe of $\mathcal{E}_2(\xi)$, $\kappa_2 \approx 0$. Motivated by this WKB behavior we approximate the argument of the electron $Z'$ function (the ion contribution is negligible in this limit) by its local value, i.e.,

$$Z' \left( \frac{3}{2} \frac{\omega_3}{\omega_s^2} \right) \approx Z'(s),$$

where

$$s \equiv \frac{1}{2} \frac{3 \omega_3}{\omega_s^2} L.$$  

Thus, from Eq. (23),

$$\varepsilon(\omega_3, \kappa) \approx 1 - \frac{3 \omega_3 L}{4 \omega_s^2} Z'(s).$$

Substituting Eq. (39) in the idler field, Eq. (21) yields, in the limit $L \to \infty$,

$$\mathcal{E}_3(\kappa) = \frac{S_3(\kappa)}{\varepsilon(\omega_3, \kappa)} \approx \frac{2 \mathcal{E}_3(\omega_3)}{\varepsilon(\omega_3, \kappa)} = \frac{2 \mathcal{E}_2}{1 - \frac{3 \omega_3 L}{4 \omega_s^2}} Z'(s).$$

The nonlinear source term $S_3(\xi)$ in Eq. (24) can then be simplified to the form

$$S_3(\xi) \approx -\frac{|\mathcal{E}_2|^2 |\mathcal{E}_1(\xi)|^2}{\omega_2} Z'(s).$$

The effective nonlinear damping coefficient $\gamma_{NL}$ associated with idler dissipation is approximately given by

$$\gamma_{NL}(\xi) = -p_1 |\mathcal{E}_2|^2 |\mathcal{E}_1(\xi)|^2 \text{Im} Z'(s) \left( 1 + \frac{2 \omega_3}{3 \omega_s L} \right) \approx p_2 |\mathcal{E}_2|^2 \left( \frac{2 \omega_3}{3 \omega_s L} \right)^2,$$

in the neighborhood of the peak of the field $\mathcal{E}_2(\omega_2)$ at $\xi \approx 2 \omega_3$, with $\mathcal{E}_2$ being the peak amplitude. The corresponding fractional power per unit area transferred from the high-frequency wave to the lower-frequency wave, and the plasma is then given by

$$\frac{\Delta P_1}{P_1} = 1 - \exp \left( -\frac{\gamma_{NL}(\xi)}{\sqrt{|\xi|^2}} \right).$$

For small values of $\gamma_{NL}$, using Eq. (43), we can then write

$$\frac{\Delta P_1}{P_1} \approx p_2 |\mathcal{E}_2|^2 \left( \frac{3 \pi \omega_3}{2 \omega_s L} \right) \left( 1 + \frac{2 \omega_3}{3 \omega_s L} \right) |\mathcal{E}_1(\xi)|^2 \frac{\omega_0}{\omega_2},$$

where $\xi_0$ is the effective width of the interaction region in this model and $P_j$ is the unperturbed power flux of the high-frequency waves, given by (for both waves)

$$P_j = \frac{\omega_j |E_0|}{8\pi} (j = 1, 2).$$

To estimate the amplification of the lower-frequency wave we note that the power transfer from the external pump to the field $\mathcal{E}_2(\xi)$ occurs where the field is resonant, i.e., within the first lobe of the Airy pattern. Consequently, we expect that the process of plasmon transfer near cutoff (i.e., near plasma resonance) simply acts as an additional pump whose amplitude can be found by manipulating $S_2(\xi)$ to be $S_2(\xi = \xi_2)$. With the approximation $\kappa_2 \approx 0$ discussed earlier, the coupling term $S_3(\xi)$ in Eq. (25) becomes

$$S_2(\xi = \xi_2) \approx \frac{2 \mathcal{E}_3(\omega_3)}{\varepsilon(\omega_3, \kappa)}.$$
with the unity term representing the contribution of the external pump at frequency \( \omega_2 \). Since in the large \( \delta_3 \) limit, \( \mathcal{E}_1(\xi) \) is well represented by its WKB form, i.e., \( \left| \mathcal{E}_1(\xi) \right|^2 \approx \pi/\sqrt{\xi_2} \), the amplification factor becomes 
\[ A = 1 + \alpha \]
with \( \alpha \) given by
\[ \alpha = -p_1 \tau \left( \frac{3\pi}{2k_A L} \right) \Im \left( \mathcal{E}_2^{\max} \right). \]

Thus, for small \( \alpha \), nonlinear damping of the higher-frequency wave at \( \omega_1 \) results in a fractional power transfer to the lower-frequency wave at \( \omega_2 \), given by
\[ \frac{\Delta P_2}{P_2} = -p_1 \tau \left( \frac{6\pi}{k_A L} \right) \Im \left[ \mathcal{E}_2^{\max} (\xi_2) \right]. \]

Finally, to compute the idler dissipation, we write
\[ P_3(\delta_3) = \frac{1}{2} \Re \int dz E_{3/4}^3, \]
and use Eq. (40) with \( \delta_3^3 = \delta_3 \) to obtain
\[ \frac{P_3}{P_1} = p_2 \tau \left( \frac{3\pi}{2k_A L} \right) \left| \mathcal{E}_2^{\max} (\xi_2) \right| \Delta \xi_3, \]
where \( \Delta \xi_3 \) is the typical width of the idler field structure. The power transferred to the idler field is smaller by a factor of \( \delta_3 / k_A L = \omega_3 / \omega_1 \) compared to the power carried by the primary fields.

Note that the power lost from the high-frequency field \( \mathcal{E}_1 \), as given by Eq. (45), includes both the power transferred to the field at frequency \( \omega_3 \) and the power dissipated through the idler field. The term proportional to \( \delta_3 / k_A L \) is the dissipation. The dissipation can be considerably smaller (by the factor \( (1 - 2\delta_3 / \omega_1) \)) than the plasmon transfer.

The simple analytical models presented in this section are useful because explicit formulas can be obtained for the various power transfer processes in terms of the scaled parameters of the problem. This provides a deeper physical insight as well as an independent check of the numerical studies, to be discussed in Sec. V. Of course, in an exact calculation it would be rigorously found that the loss of power through the plasmon transfer term obtained in Eq. (45) would be identical to the power increase in the lower-frequency wave, i.e., \( P_{1/2} (50) \). In the approximation models discussed here, however, these two results are not identical, but differ only by a numerical factor of order unity. The discrepancy arises from the method used to estimate the effective amplitude and width of the localized field used to model the first Airy lobe of \( \mathcal{E}_2 \). A similar comment applies to the comparison between the idler dissipation term (proportional to \( \delta_3 / k_A L \)) contained in Eq. (45), and the corresponding Joule-heating calculation leading to Eq. (53). If a closure is desired for further applications of these formulas, then it would be appropriate to equate the different expressions to determine the consistent values for the effective width and amplitude of the modeled \( \mathcal{E}_2 \) field.

In arriving at the expression given in Eq. (41) for the nonlinear source term \( S_1(\xi) \), it has been assumed that the ion contribution to the \( Z' \) function is negligible, as is appropriate for \( \delta_3 \) large. For completeness, we point out that the general form of Eq. (41) is
\[ S_1(\xi) = -\frac{\left| \mathcal{E}_2 \right|^2 \mathcal{E}_1(\xi)}{\omega_2} Z'_e(s) \times \left( \frac{1 - (3k_A L/4\delta_3) \left[ 2 + Z'_e(s) \right]}{1 - (3k_A L/4\delta_3) Z'_e(s)} \right), \]
where \( Z'_e(s) \) refers to the electron contribution and \( Z'_e(s) \) to the sum of the electron and ion contributions, as defined by Eq. (23).

From Eq. (54) it is then possible to calculate that as \( \delta_3 \) is reduced, the contribution from the \( \Im Z'_e \) becomes very small, while the resistive contribution from the beat resonance of the idler field with the ions begins to play a role. The picture that emerges as \( \delta_3 \to 0 \) from the large \( \delta_3 \) side is that the electrons become adiabatic, while the ions go from being cold to contributing a peak in the dissipation to eventually becoming adiabatic as well, and the conventional ponderomotive result given by Eq. (33) is recovered directly from Eq. (54).

IV. CONSERVATION LAWS

It is important to identify the conservation laws for this problem. These equations provide a clearer understanding of the physical processes involved, and also give an independent check of the numerical procedure used to solve the field equations.

Multiplying Eq. (7) with \( q(z) \) and taking the imaginary part results in the energy conservation equation for the individual Langmuir waves,
\[ \frac{\partial E_j^3(z)}{\partial z} + F_j^3(z) + P_j^3(z) + P_j^{NL}(z) = 0, \]
where
\[ W_j = -\frac{3\omega_j}{k_B} \Im \left( \frac{E_j^3 \partial E_j^3}{\partial z} \right), \]
\[ P_j^{nl} = \omega_j E_0^{\ast} \Im (E_j), \]
\[ P_j^2 = \Gamma_j \omega_j |E_j|^2, \]
\[ P_j^{NL} = \frac{1}{2} \Re (E_j^3 P_j^{NL^3}). \]
Physically, \( W_j \) represents the wave-power flux, \( P_{jn}^{\text{in}} \) is the input power density provided by the external source, \( P_{jn}^{\text{coll}} \) is the power density dissipated by collisions, and \( P_{jn}^{\text{NL}} \) is the nonlinear power density exchange at frequency \( \omega_j \). The latter is associated with the beat-coupling process.

Substituting Eqs. (9) and (10) for \( j_{n_2}^{\text{NL}}(z) \) in Eq. (59) yields

\[
P_{1n}^{\text{NL}}(z) = -\frac{e}{m\omega_2} \frac{k_0^2}{16\pi} \Re \left[ \frac{E_1^*(z)E_2(z)}{\omega_2} \right] \mathcal{S}^{-1}(\tilde{\Phi}) \]

\[
 = -\frac{\omega_2}{\omega_1} \frac{\partial}{\partial z} \left[ \left( \frac{1 - \omega_2^2}{L} \right) E_2(z) \right] \mathcal{S}^{-1}(\tilde{\Phi}^*).
\]

(60)

\[
P_{2n}^{\text{NL}}(z) = -\frac{e}{m\omega_1} \frac{k_0^2}{16\pi} \Re \left[ \frac{E_2^*(z)E_1(z)}{\omega_1} \right] \mathcal{S}^{-1}(\tilde{\Phi}^*) \]

\[
 = -\frac{\omega_1}{\omega_2} \frac{\partial}{\partial z} \left[ \left( \frac{1 - \omega_1^2}{L} \right) E_1(z) \right] \mathcal{S}^{-1}(\tilde{\Phi}^*).
\]

(61)

One can also derive the conservation equations for the entire system. Adding Eq. (55) for \( \omega_1 \) and \( \omega_2 \) yields

\[
\sum_{j=1}^{2} \left( \frac{\partial}{\partial z} \frac{W_j}{\omega_j} + P_{jn}^{\text{in}}(z) + P_{jn}^{\text{coll}}(z) + P_{jn}^{\text{NL}}(z) \right) = 0.
\]

(63)

Also, dividing Eq. (55) by \( \omega_j \), and summing, results in

\[
\sum_{j=1}^{2} \left[ \frac{\partial}{\partial z} \left( \frac{W_j(z)}{\omega_j} \right) + \Gamma_j \left( \frac{|E_j|^2}{8\pi} + \frac{E_0^2}{8\pi} \right) \text{Im}(E_j) + \frac{P_{jn}^{\text{NL}}(z)}{\omega_j} \right] = 0.
\]

(64)

Two separate processes, namely, plasmon transfer from the higher-frequency wave to the one at lower frequency, and idler dissipation contribute to the nonlinear power exchange term \( P_{jn}^{\text{NL}}/\omega_j \). Using Eqs. (60) and (61), it is easy to demonstrate that the plasmon transfer terms exactly cancel out, and we are left with the nonlinear power dissipation term

\[
\sum_{j=1}^{2} \frac{P_{jn}^{\text{NL}}}{\omega_j} = \frac{e\omega_3}{m\omega_1\omega_2} \frac{k_0^2}{16\pi} \text{Im} \left[ \frac{E_1^*(z)}{\omega_2} \frac{\partial}{\partial z} \left( \frac{1 - \omega_2^2}{L} \right) E_2(z) \right] \times \mathcal{S}^{-1}(\tilde{\Phi}^*) \]

\[
 = \frac{\omega_3}{\omega_2} \frac{k_2(z)}{\omega_2} \times \mathcal{S}^{-1}(\tilde{\Phi}^*).
\]

To gain insight into the meaning of Eq. (65), it is useful to evaluate the right-hand side in the WKB limit, in which, sufficiently far from the resonance point of both waves, we can approximate

\[
\frac{i}{\omega_2} \frac{\partial}{\partial z} \left[ \left( 1 - \frac{z^2}{L} \right) E_2(z) \right] \approx \frac{k_2(z)}{\omega_2} E_2(z),
\]

(66)

\[
\frac{i}{\omega_1} \frac{\partial}{\partial z} \left[ \left( 1 - \frac{z^2}{L} \right) E_1(z) \right] \approx \frac{k_1(z)}{\omega_1} E_1(z),
\]

(67)

where \( k_1(z) \) and \( k_2(z) \) are the WKB wave numbers of the fields at frequencies \( \omega_1 \) and \( \omega_2 \), respectively. In this limit, Eq. (65) reduces to

\[
\sum_{j=1}^{2} \frac{P_{jn}^{\text{NL}}}{\omega_j} = \frac{1}{m\omega_1 \omega_2} \frac{|E_1|^2}{8\pi} \frac{|E_2|^2}{8\pi} \frac{\omega_3}{\omega_1} \Im Z^*_e \left( \frac{1}{\sqrt{2(k_1 - k_2)^2} \omega_3} \right).
\]

(68)

Note that the nonlinear dissipation depends directly upon \( \text{Im} Z^*_e \) and is proportional to \( \omega_3 \).

V. NUMERICAL RESULTS

The coupled differential equations (19)–(21) are solved numerically using an iterative procedure. Typically 256 points with grid spacing \( \Delta \xi = 0.156 \) are used. For the results reported here the collision frequency is set to zero in order to accentuate the intrinsic dissipation associated with nonlinear Landau damping. The pump amplitudes \( E_0 \) and \( E_0^2 \) are taken to be equal in these surveys, and a plasma with \( T_e/T_i = 1, k_0L = 100 \) is considered. This reduces the total number of parameters to the two essential quantities: the nonlinear coupling parameter \( p = (k_0L)^2 \frac{E_0^2}{24\pi n_0 T_e} \) and the scaled frequency separation \( \delta_3 = (1 - \omega_2/\omega_1) k_0L \).

To ensure convergence of the Fourier transforms, a pseudo-Landau damping term is incorporated that acts (in configuration space) at \( \xi > 25 \), which is far from the interaction region of interest to our study. A Green's function technique is used to transform the differential equations in configuration space (19) and (20) into integral equations for the fields \( E_1(z) \) and \( E_2(z) \). The integral equations are then solved by iteration. In the first trial, the source for the idler field in Eq. (21) is calculated using the unperturbed driven Airy fields \( \sum_{j=1}^{2} \frac{E_j(z)}{\omega_j} \) at frequencies \( \omega_1 \) and \( \omega_2 \). The idler field is then computed in \( k \)-space and inverted to obtain \( E_3(z) \), which is used in the integral equation to calculate the modified \( E_1(z) \) and \( E_2(z) \). A weighted average of the previous two iterations is used as a new estimate and the process is repeated. In a typical run, four to ten iterations are found to be sufficient to obtain convergence. However, as the pump amplitude increases, convergence of the solutions requires more iterations. Eventually, for \( p > 0.60 \), convergence becomes a problem; so we limit our study to smaller amplitudes. It should be noted, however, that the nonlinear Landau damping interaction is essentially over before the large \( p \) regime is reached. Hence, the modification of the density profile by the ponderomotive force is not important for the regime under study. As shown by Morales and Lee,\(^{11}\) this effect becomes significant for \( p > 1 \).

The scaled amplitudes of the unperturbed electric fields resonantly excited at frequencies \( \omega_1 \) and \( \omega_2 \) are illustrated in Fig. 4 for \( \delta_3 = 2 \). These fields are driven by an external capacitor-like pump \( (k_0 = 0) \). The modulation in the amplitude is due to the interference of the uniform-
pump field and the resonantly excited Langmuir wave. In the absence of Landau damping, these field structures are described by the driven-Airy waveforms, \( \pi(G(z) - iA(z)) \).

The modifications produced by nonlinear Landau damping on the fields shown in Fig. 4 are exhibited in Figs. 5 and 6 for a value of \( p=0.2 \). The higher-frequency field \( \mathcal{E}_1(\xi) \) undergoes enhanced damping, while the lower-frequency field \( \mathcal{E}_2(\xi) \) is amplified simultaneously with acceleration of the background plasma electrons through idler dissipation at \( \omega_3 \). Figures 7 and 8 illustrate the effect of increasing the frequency separation to a value \( \omega_3=4 \). It is seen in Fig. 7 that the onset for the damping of the high-frequency wave occurs at a larger value of \( \xi \) than in Fig. 5, because the Airy peaks are farther separated for a larger value of \( \omega_3 \). In both cases, however, most of the energy transfer occurs in the region of the Airy peak of the low-frequency wave.

The Fourier spectrum of the idler field associated with Figs. 5 and 6 is shown in Fig. 9. The presence of negative and positive \( k \) contributions suggests that the idler is a partially localized structure in configuration space, as is illustrated in Fig. 10. The idler field has a maximum at a location determined by the overlap of the first lobe of the field \( \mathcal{E}_2(\xi) \) with the \( \mathcal{E}_1(\xi) \) field. The idler field is largest at this location because the nonlinear source term

\[ \text{FIG. 4. Spatial dependence of the scaled amplitude of the unperturbed (linear) resonant fields for } \tilde{\omega}_3=2.0 \text{ and } E_0=E_0. \]

\[ \text{FIG. 5. Spatial dependence of the scaled amplitude of the high-frequency field at } \omega_2 \text{ for the same parameters as Fig. 5. The solid curve is the unperturbed (linear) field amplitude and the dashed curve is the nonlinearly modified amplitude showing enhancement from plasmon transfer.} \]

\[ \text{FIG. 6. Spatial dependence of the scaled amplitude of the high-frequency field at } \omega_3 \text{ for } p=0.2 \text{ and } \omega_3=4.0. \]
$S_j (j=1,2)$ is proportional to the product of the individual wave amplitudes.

Figure 11 shows the spatial dependence of the power dissipated in the plasma through beat resonance of the idler field with slow electrons for $\omega_3=2.0$. Again, the dissipation is maximum at a spatial location determined by the peak amplitude of $g_2$. The phase velocity of the idler is approximately $(3M/mkAL)^{1/2}$ times larger than the ion thermal velocity, and for the parameter regime studied, this ratio is 7. Hence, ions do not participate in the idler dissipation, which is entirely due to plasma electrons.

Since the scaled pump amplitude $p_j$ determines the strength of the nonlinear interaction, it is expected that the relative enhancement of $g_2$ and damping of $g_1$ increases as $p_j$ increases. Figure 12 shows the field amplitude of the higher-frequency wave $g_1$ for $\omega_3=2.0$ and $p_1=0.10$, 0.20, and 0.40. For $p_1=0.40$, we see that most of the wave energy from $g_2$ has been transferred to the lower-frequency wave, and the interaction is over before $\xi=10$. The remaining field amplitude beyond $\xi=10$ is too small to induce a significant nonlinear interaction.

Figure 13 shows the dependence of the idler dissipation on the frequency difference between the pump fields. The open triangles are the self-consistent numerical results. The solid and the dashed straight lines represent two different predictions based on Eq. (53). The two estimates differ in the approximation used for $|g_1|^2$. The dashed line is obtained by estimating $|g_2|^2 \Delta P_3$ as the area under the
FIG. 12. Dependence of plasmon transfer on pump strength for various coupling parameter values at $\tilde{\omega}_2 = 2.0$. The spatial dependence of the scaled amplitude of the field at frequency $\omega_1$ is shown for $p=0.10$, 0.20, and 0.40, respectively.

The curve $|\mathcal{G}_2|^2$ from negative infinity to the location of the first minimum after the Airy-lobe peak. The solid line is for $|\mathcal{G}_2|^2 \Delta \xi_2$ estimated as the area under the principal lobe of $|\mathcal{G}_2|^2$.

In the limit of $\omega_1 \rightarrow 0$, the two primary waves merge into one, and the interaction can be viewed as a ponderomotive process, as shown in Sec. III. In this case, the idler field generates a localized density depletion and the power dissipation to the plasma goes to zero, consistent with the prediction of Eq. (53).

The self-consistent numerical value obtained for the asymptotic fractional power extracted from the high-frequency field for $\tilde{\omega}_3 = 4$, and $p_j = 0.2$ (as shown in Fig. 7) is 0.64. For the two estimates of $|\mathcal{G}_2|^2 \Delta \xi_2$, the corresponding result from Eq. (45) is 0.39 and 0.43. The discrepancy between the fractional power loss predicted by Eq. (45) and the numerical results can be accounted for by noting that the model only estimates the damping across the principal lobe of $\mathcal{G}_2$. The numerical power loss at the end of the first Airy lobe of $\mathcal{G}_2$ is 0.47, a result in much better agreement with the model estimates.

Figure 14 illustrates the spatial dependence of the power flows associated with nonlinear Landau damping of the wave at frequency $\omega_1$ for $\tilde{\omega}_3 = 2$, $p = 0.4$, in the absence of collisions. The curve labeled $W_1$ is the wave power flux [Eq. (56)]. The curve labeled $Q^{in}$ is the integrated input power of the pump at $\omega_1$ [Eq. (69)], and $Q^{nl}$ is the nonlinear power exchange [Eq. (70)]. The flat line shows the integral of Eq. (55), given by $W_1 + Q^{in} + Q^{nl}$, as a ratio to the pump input power $P_1$ [Eq. (46)]. Its value is smaller than $10^{-3}$.

FIG. 13. Dependence of the total power absorbed by the plasma on scaled fractional frequency separation $\tilde{\omega}_3 = (\omega_3 - \omega_2)k_xL/\omega_2$ for $p = 0.2$. Solid and dashed straight lines are the analytical approximation [Eq. (33)] obtained for $\tilde{\omega}_3 > 2\Delta \omega_m$ using two different estimates for the region of interaction. Triangles are self-consistent numerical results.

FIG. 14. Spatial dependence of the power flows associated with nonlinear Landau damping of the higher-frequency wave at $\omega_1$ for $\tilde{\omega}_3 = 2$, $p = 0.4$, in the absence of collisions. Here $W_1$ is the wave power flux [Eq. (56)], $Q^{in}$ is the integrated input power of the pump at $\omega_1$ [Eq. (69)], and $Q^{nl}$ is the nonlinear power exchange [Eq. (70)]. The flat line shows the integral of Eq. (55), given by $W_1 + Q^{in} + Q^{nl}$, as a ratio to the pump input power $P_1$ [Eq. (46)]. Its value is smaller than $10^{-3}$.
pump and the traveling Langmuir wave. This is a linear effect unrelated to nonlinear Landau damping. The nonlinear power exchange (dominated by the plasmon transfer process) is seen to rise steeply, along with a strong reduction in the $\omega_1$ wave power flux in the region of the Airy peak of the low-frequency wave ($\xi \approx 5.0$).

VI. CONCLUSION

The present study provides a specific example of a self-consistent investigation of a kinetic nonlinearity in a nonuniform plasma. The solution to the present problem is facilitated by the separation of fluid-like properties from the fully kinetic features. In a nonuniform medium the fluid-like properties are described by a differential equation in configuration space, while the kinetic features require a differential equation in Fourier space. This technique can be viewed as the nonuniform medium generalization of the powerful method used to treat beam nonlinearities in uniform media, and it may prove quite valuable in attacking a wide class of problems.

It is found that although the general picture of beat-wave–particle interaction developed for uniform plasmas is not qualitatively changed, the plasma nonuniformity introduces important modifications in the waveforms that invalidate the straightforward WKB extrapolation of the results. The situation is summarized in Fig. 13, where it is seen that the power absorbed by slow electrons increases linearly with frequency separation. The plasma nonuniformity introduces a critical frequency separation $(\Delta \omega/\omega)_c = 1.8/k_A L$ determined by the spatial width of the first Airy lobe. For frequency differences larger than the critical value, the interaction is essentially determined by the beat of a relatively short-wavelength Langmuir wave at $\omega_1$ and a nearly uniform (i.e., $k_2 \approx 0$) electric field at $\omega_2$. This implies that the resonant velocity associated with the idler field is $(\omega_1 - \omega_2)/k_1$, and not the usual group velocity. Of course, the monotonic increase in electron absorption with increasing frequency separation is eventually limited by linear Landau damping of the higher-frequency field at $\omega_1$. This effect is expected to set in when the frequency separation causes the resonant fields to be spaced by one Landau damping length of the higher-frequency wave. This limits the maximum frequency separation to $(\omega_1 - \omega_2)/\omega_1 \leq \sqrt{3}$.

It has been found that the power absorbed by slow electrons is a factor of $(1 - \omega_2/\omega_1)$ smaller than the power transferred from the high to low-frequency wave. However, as Eq. (41) shows, the dissipative response of the slow electrons is essential to the effective plasmon transfer process. Strong nonlinear Landau damping of the high-frequency wave is found to occur at pump amplitudes $(p \sim 0.2)$ below the level at which ponderomotive modifications in the profile play a major role $(p \sim 1)$. The value of pump-strength parameter $p$ for which strong nonlinear Landau damping occurs is also smaller than the effective value $(p \geq 4)$ used by White et al. in assessing the regime in which the parametric decay instability is excited in laser-plasma interactions.

For frequency differences smaller than the critical value, the nonlinear Landau damping process has been shown to provide a generalization of the ponderomotive interaction to nonzero frequencies. The usual adiabatic result is recovered for $\omega_2 = 0$, but, as is indicated by Eq. (54), resistive ion contributions are present for $\omega_2 \neq 0$.

The features described in this study may play a role in resonant absorption experiments in which a frequency spread is present. The two characteristic signatures of the nonlinear process are acceleration of slow electrons, and a spectral shift to low frequencies. Since, under certain conditions, the accelerated electrons can excite secondary waves through sideband instabilities and Ærøenkov emission, these signals could be misinterpreted as arising from parametric instabilities.

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