Modeling the propagation of whistler-mode waves in the presence of field-aligned density irregularities

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We present a numerical study of propagation of VLF whistler-mode waves in a laboratory plasma. Our goal is to understand whistler propagation in magnetic field-aligned irregularities (also called channels or ducts). Two cases are examined, that of a high-frequency (\(\omega > \Omega_e/2\)) whistler in a density depletion duct and that of a low-frequency (\(\omega < \Omega_e/2\)) whistler in a density enhancement. Results from a numerical simulation of whistler wave propagation are compared to data from the UCLA Los Angeles Physics Teachers Alliance Group plasma device and whistler propagation in pre-existing density depletion and density enhancement ducts is demonstrated.

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I. INTRODUCTION

Over fifty years ago, Storey\(^1\) proposed that whistler waves can be guided along geomagnetic field lines by plasma density irregularities or “ducts.” Satellite observations\(^2–4\) have confirmed that whistlers can indeed be guided by field-aligned tubes of density enhancement (“density crests” or high-density ducts (HDDs)), and also by tubes of density depletion (“troughs” or low-density ducts). There is also a long history of theoretical studies of the ducting phenomenon. Since the diameter of magnetospheric ducts was thought to be usually much larger than the wavelength of the whistlers, nearly all of the theoretical works\(^5–8\) have been based on ray tracing and other techniques of geometrical optics. This body of theoretical work shows that whistlers of any frequency can be guided by low-density ducts, and that whistlers of frequency \(\omega < \Omega_e/2\), where \(\Omega_e\) is the electron gyrofrequency, can also be guided by HDDs.

Density irregularities are now known to occur on a variety of spatial scales, from 0.1 m to 100 km.\(^9\) Ray-tracing methods are not valid when density variations occur on spatial scales which are comparable to the wavelength, so the most frequently used tool for studying magnetospheric whistler (\(\lambda \sim 10\) km) propagation cannot be relied upon in many cases. In these circumstances, it is necessary to solve Maxwell’s equations directly to obtain a full-wave solution.

Ducting of whistlers is the most commonly cited effect of density irregularities on magnetospheric whistlers, but observations indicate that it may not be the only one. For example, a recent study by Haque et al.\(^10\) used data from the Cluster spacecraft to demonstrate an association between the occurrence of lower-band whistler chorus and density depletion ducts, and it has been suggested by Morgan et al.\(^11\) that density depletions may play a role in the formation of auroral hiss. Despite decades of study into the propagation of whistlers in large scale density irregularities, there is much yet to be understood about the interaction of whistler-mode waves and density irregularities, especially at scales comparable to and smaller than the whistler wavelength.

II. EXPERIMENT

In this study, we model whistler waves observed in experiments conducted at the using the UCLA Basic Plasma Science Facility using the Los Angeles Physics Teachers Alliance Group (LAPTAG) plasma device.\(^12\) The LAPTAG device has a 30 cm diameter plasma that is 150 cm long; data are taken in a plane along the axis of the device in a rectangular region 71 cm long and either 18 cm or 22 cm wide (the size varies between experiments, but the grid spacing is 1 cm along the axis and 0.35 cm off-axis). The LAPTAG afterglow plasma has a temperature of 0.4 eV, and axial magnetic field of 95 G in these experiments, and an electron density in the range \(10^{10}–10^{11}\) cm\(^{-3}\); these values do not change appreciably during the experimental timescale of 40 ns. Signals are introduced to the plasma by using a 4 cm loop antenna to transmit a phase-locked tone burst at a single frequency. Magnetic field measurements are obtained using a three-axis B-dot loop with data output every 0.4 ns. Note that the same probe is used for taking every measurement, with the experiment being repeated at each spatial location, of which there are either 3621 (18 cm by 71 cm) or 4331 (22 cm by 71 cm). Density ducts are created by placing a small disk of metal in front of the plasma source, thus, preventing electrons from streaming in along the field lines. Ions remain outside of the ducts to preserve charge neutrality.

Experiments involving two different types of plasma configuration are considered, one type having a plasma with only gradual spatial variations in density and the other type having a plasma with a localized region of decreased density. For each type, both high- \((\omega > \Omega_e/2)\) and low-frequency...
(ω < Ωce/2) whistlers are considered. The measured plasma density profiles for these cases are shown in Figures 1 and 4 (note the magnitude of the density profiles is obtained from a measurement of the ion saturation current calibrated with wave dispersion measurements as discussed in Sec. IV C).

III. MODEL

Propagation of whistler-mode waves in laboratory plasma can be described with a set of so-called electron magnetohydrodynamics (EMHD) equation. The EMHD model assumes that ions are immobile and the electrons are considered as a cold fluid, carrying current, j = −en_i v. Because ions are immobile and the plasma is assumed to be quasineutral (n_e ≈ n_i ≡ n), the plasma density in the model does not change in time, and the model consists of the electron momentum equation (for the electron fluid velocity, v), and Maxwell’s equations for the electric, E, and magnetic, B, fields

\[ \frac{\partial v}{\partial t} + (v \cdot \nabla)v = -\frac{e}{m_e}(E + v \times B) - \nu v, \quad (1) \]

\[ \nabla \times B = \mu_0 j, \quad (2) \]

\[ \nabla \times E = -\frac{\partial B}{\partial t}. \quad (3) \]

The coefficient ν in Eq. (1) is the electron collision frequency (primarily electron-neutral). The displacement current is neglected in the Ampère’s law [Eq. (2)]. This assumption is known in the literature as a “quasi-longitudinal” approximation of EMHD, which is valid when the wave angular frequency, ω, satisfies conditions ω_h < ω < ω_e ≪ ω_pe. Here ω_h is the lower hybrid frequency; ω_e is the electron gyrofrequency; and ω_pe is the electron plasma frequency.

When the background plasma and the magnetic field are homogeneous in space and collisions are not important, the linearized set of Eqs. (1)–(4) yields the quasi-longitudinal whistler mode dispersion relation

\[ k^2 - \frac{\omega_{pe}^2}{\omega}k||k + \frac{\omega^2}{c^2} = 0, \quad (4) \]

where \( k^2 = k_0^2 + k_\perp^2 \) and the notations \( || \) and \( \perp \) denote the directions parallel and perpendicular to the background magnetic field, respectively.

Although relation (4) is rigorously correct only within regions where the ambient plasma and magnetic field are homogeneous, it can also be regarded as an approximate local relation even in inhomogeneous regions, which provides a simple intuitive picture of whistler trapping by the density inhomogeneities, a useful complement to our numerical analysis of the complete set of Eqs. (1)–(3).

A. Basics of whistler ducting

The dispersion relation (4) is usually analyzed separately for low-frequency and high-frequency cases. A high-frequency whistler is defined as having ω > Ωce/2 while a low-frequency whistler is defined as having ω < Ωce/2. This separation is motivated by a topological change in the whistler index of refraction surface which occurs at ω = Ωce/2. Ray theory predicts that this topological change should be accompanied by a change in the behavior of ducted whistlers.

Streltsov et al. used Eq. (4) to derive an expression for the perpendicular component of the wave vector. Using \( n_1 = n_i k_0^2 / \mu_0 e^2 (\Omega_{ce}/2\omega)^2 \), this expression may be written as

\[ k_\perp = k || \left( \frac{\Omega_{ce}^2}{4\omega^2} \left( 1 \pm \sqrt{1 - \frac{n_1^2}{n_0^2}} \right) - 1 \right)^{1/2}. \quad (5) \]

Let us denote the solutions to Eq. (5) as \( k_\perp^\pm \). For ω < Ωce/2, it is possible for both \( k_\perp^\pm \) to be real, whereas only \( k_\perp^+ \) can be real when ω > Ωce/2. The criteria for existence of these solutions are dependent upon two characteristic plasma densities, \( n_0 = n_i k_0^2 / \mu_0 e^2 (\Omega_{ce}/\omega - 1) \) and the previously defined \( n_1 \) (as we will discuss, \( n_0 \) and \( n_1 \) are the cutoff densities for high- and low-frequency whistlers, respectively,

![FIG. 1. Experimental density profiles used in simulation of high-frequency whistlers. Left: uniform density profile. Right: duct density profile.](image-url)
and in the low-frequency case \( n_0 \) is the density at which \( k_\perp = 0 \).

Regardless of frequency, neither of \( k_\perp^\pm \) are real if \( n > n_1 \). When \( \omega < \Omega_c/e/2 \), both \( k_\perp^\pm \) are real if \( n_0 < n < n_1 \), and only \( k_\perp^+ \) is real if \( n < n_0 \). When \( \omega > \Omega_c/e/2 \), only \( k_\perp^+ \) may be real, and only if \( n < n_0 \). Note, however, that the evanescence of the low-frequency whistler is due to the introduction of a small imaginary part while the evanescence of the high-frequency whistler is due to an entirely imaginary wavenumber. Consequently, it would be expected that low-frequency whistlers would be better able to escape from a depletion duct, especially if that duct is shallow and/or bounded by density enhancements of limited spatial extent.

1. Ducting in a density depletion

These considerations suggest the following scenarios for the propagation of whistlers in a density depletion duct. In the high-frequency case (\( \omega > \Omega_c/e/2 \)), the wave may propagate freely so long as \( n < n_0 \). The wave has a turning point where \( n = n_0 \), and is evanescent beyond. It should be noted that, in this case, ray theory predicts that the wave normal will tend towards regions of lower density, thus, enhancing the wave’s confinement to the depletion duct. On the other hand, in the low-frequency case (\( \omega < \Omega_c/e/2 \)) the situation is more complicated. Rather than becoming evanescent, the whistler wave solution bifurcates at \( n = n_0 \) and does not become evanescent until \( n = n_1 \). If the density profile within the duct varies smoothly from some minimum value \( n_{\text{min}} \) up to \( n_1 \), there is a region where \( n_0 < n < n_1 \) and the two solutions can exist simultaneously; if \( n_{\text{min}} < n_0 \), this region will only comprise a fraction of the duct.

Based on these considerations, it is expected that whistler waves can propagate in a depletion duct regardless of frequency so long as \( n < n_0 \). In the low-frequency case, the waves may propagate into regions of higher density, but when \( n_0 < n < n_1 \), the plasma permits the existence of a second propagating whistler mode wave. This sort of ducting was suggested in many early theoretical studies, summarized by Helliwell and also has been convincingly demonstrated in a series of laboratory experiments. Depending on the depth and transverse extent of the duct, it is possible for this wave to be confined to a single density gradient in one half of the duct. This so-called one-sided ducting has been discussed by Helliwell and Inan and Bell and was later demonstrated in simulations by Streltsov et al. Note that it is also possible for large-amplitude whistlers to create their own density ducts, but we are limiting our consideration to waves whose amplitudes are too small for this to occur (this is also true of the experimental results discussed below).

2. Ducting in a density enhancement

It is not immediately obvious from the discussion in the Sec. IIIA 1 that whistlers should also be ducted in regions of enhanced density. However, the apparent guiding of whistlers between conjugate points on geomagnetic field lines and spacecraft observations of the associated plasma density structure are highly suggestive of such ducting actually taking place. Conceptually, the biggest impediment to high-density ducting is the absence of a cutoff density; if the wave is able to propagate in the enhanced region, it should also be able to propagate in the surrounding regions of lower density. However, there are at least three situations in which this is not the case. First, as discussed in Sec. IIIA 1, for low-frequency whistlers there is a second whistler mode which exists only when \( n_0 < n < n_1 \). If the density enhancement is of sufficient magnitude, then a cutoff for this mode will exist where \( n = n_0 \) and it is possible for this wave to propagate loss-free in the high-density region (it is loss-free because \( k_\perp^+ \) is purely imaginary for \( n < n_0 \). If the density is below \( n_0 \), it is still possible for the wave to be ducted, but it is now subject to losses via coupling to waves outside the duct. As noted by Helliwell, low-frequency whistler ray paths will tend to be directed towards regions of high density. As a consequence of this, whistlers whose ray angle is within a certain (density-dependent) range will remain confined to the duct while others (with more oblique angles) would not. This manner of ducting is analogous to the propagation of visible light signals in a graded-index fiber optic cable. It is not obvious that this mechanism will be effective for the propagation of signals in a narrow duct, however, since geometrical optics is not valid in this situation and, moreover, the wave would have to be almost perfectly field-aligned even if it were. Finally, it was noted by Streltsov that a whistler could propagate loss-free in a high-density duct if the width of the duct were an integer number of wavelengths. In this case, the whistler wave external to the duct would have a vanishingly small amplitude and associated energy leakage from the duct would be minimal.

IV. SIMULATION

A. Numerical implementation

Using relation \( \mathbf{j} = -\mathbf{v} \times \mathbf{E} \), Eqs. (1)–(3) can be used to obtain the following vector equations for \( \mathbf{E} \), \( \mathbf{v} \), and \( \mathbf{B} \)

\[
\begin{align*}
\delta_e \nabla \times \nabla \times \mathbf{E} + \mathbf{E} &= -\frac{m_e}{e} \left( \mathbf{v} \cdot \nabla \mathbf{v} + \frac{e}{m_e} \mathbf{v} \times \mathbf{B} + \nu \mathbf{v} \right), \\
\frac{\partial \mathbf{v}}{\partial t} &= - \left( \mathbf{v} \cdot \nabla \mathbf{v} + \frac{e}{m_e} \mathbf{v} \times \mathbf{B} + \nu \mathbf{v} \right) - \frac{e}{m_e} \mathbf{E}, \\
\frac{\partial \mathbf{B}}{\partial t} &= - \nabla \times \mathbf{E},
\end{align*}
\]

where \( \delta_e = c/\omega_{pe} \) is the electron inertial length. Equations (6)–(8) are solved numerically using the finite-difference time-domain (FDTD) technique inside a rectangular, two-dimensional domain that represents a cut along the axis of the experimental device used by the LAPD group. The domain is described with 2D Cartesian coordinate system \((x,z)\), where \( z \) is the direction along the background magnetic field, \( \mathbf{B}_0 \), (which is assumed to be homogeneous) and \( x \) is the direction orthogonal to \( \mathbf{B}_0 \) and it is discretized with a uniform numerical grid with 155 × 501 nodes (\( \Delta x = \Delta z = 0.14 \text{ cm} \) for the 22 cm by 71 cm grid). The FDTD approach is implemented by approximating all spatial derivatives in Eqs. (6)–(8) with mixed second- and fourth-order finite differences. Specifically, the spatial derivatives in Eqs. (7) and (8) are
approximated using fourth-order finite differences while Eq. (6) is approximated using second order finite differences, which allows us to solve the equations using the successive over-relaxation (SOR) method with red-black ordering.20

Because iterative methods such as SOR require significantly more work than a straightforward marching method, it is desirable to minimize the number of times the solver must be used. A pure marching method such as Runge-Kutta requires only two evaluations, regardless of accuracy. Because our goal is fourth order accuracy, the choice of a predictor-corrector method requires almost four times less work than Runge-Kutta. Thus, to advance Eqs. (7) and (8) in time, we use a fourth-order Adams-Bashforth-Moulton predictor-corrector, with an Adams-Bashforth (AB) method serving as the predictor and an Adams-Moulton (AM) method serving as the corrector.21 In order to calculate values of E, B, and v at a time \( t + \Delta t \) (where \( \Delta t \) is a time step) from known values at a time \( t \), we first execute a “predictor” part of the algorithm and calculate \( B^{t+\Delta t} \) and \( v^{t+\Delta t} \) from \( E^{t} \) by using AB method applied to Eqs. (7) and (8); then we calculate \( E^{t+\Delta t} \) from \( B^{t+\Delta t} \) and \( v^{t+\Delta t} \) using Eq. (6). Next, we use \( E^{t+\Delta t} \) in the “corrector” part of the algorithm and calculate \( B^{t+2\Delta t} \) and \( v^{t+2\Delta t} \) from Eqs. (8) to (7) using AM method. Finally we calculate the new value of \( E^{t+2\Delta t} \) from \( B^{t+2\Delta t} \) and \( v^{t+2\Delta t} \) using Eq. (6).

B. Boundary conditions

It is clear from the description of this approach that boundary conditions should be specified for \( E \) only. Because we study wave propagation inside plasma surrounded by a conducting cylinder, the boundary conditions for \( E \) on these boundaries reflects the nature of the problem: The tangential components of \( E \) are equal to 0 at the boundary, and normal derivatives of the components orthogonal to the boundaries are also equal to 0.

At the initial moment of time \( (t = 0) \) all disturbed values of \( E, B, \) and \( v \) are set to zero and a wave with the frequency \( \omega \) and the perpendicular wavenumber \( k_{\perp} \) is injected from the bottom boundary of the domain \((z = 0)\), where \( E \) is specified as

\[
E_{x}(x,t) = -E_{0}D(x)\cos(k_{\perp}x)\sin(\omega t),
\]

\[
E_{y}(x,t) = E_{0}D(x)\cos(k_{\perp}x)\cos(\omega t),
\]

\[
E_{z}(x,t) = E_{0}D(x)\sin(k_{\perp}x)\cos(\omega t),
\]

(9)

Here \( D(x) = 1/\exp((x-x_{0})/x_{1})^{4} \) is a function approximating a spatially localized antenna; \( x_{0} \) is the \( x \) coordinate of the antenna, and \( x_{1} \) is the parameter defining the characteristic transverse size of the field created by the antenna (analogous to the diameter of the antenna loop used in the experiment).

Although the transverse boundaries are assumed to be perfect conductors, reflections from these boundaries would not accurately reflect the experimental system being modeled (the device is larger than the region in which measurements are taken, with a very low density plasma separating the wall from the assumed boundary in these experiments). In order to minimize the effects of spurious reflections, an absorbing layer is placed on the edges of the simulation domain. Specifically, a small constant collision frequency \( \nu_{e} \) is assumed for the majority of the domain, but in a relatively narrow region of width \( \Delta x \) near the edges, the collision frequency increases quadratically to a much larger value of \( \nu_{e} \) (with \( \nu_{e} > \omega \)). This method of absorption was discussed in detail by Streltsov et al.16

C. Background parameters

The background magnetic field is uniform throughout the entire domain, directed in the \( z \)-direction with magnitude \( B_{0} = 95 \) G. The electron cyclotron frequency in this plasma is 265 MHz. The two-dimensional relative density profile was obtained from measurements of the ion saturation current. Because of uncertainties in the probe area, conductivity of any coating, and possible sheath effects, the saturation current alone cannot be used to accurately determine the density. However, if the plasma density is measured by another means such as interferometry or using wave dispersion, then the saturation current can be calibrated and used to determine the actual density. Therefore, we used the parallel whistler wavelength and whistler mode dispersion relation (4) to estimate the density and used this to calibrate the saturation current data. The resultant density profiles are shown in Figure 1. We consider two different whistler waves, one with \( f = 90 \) MHz and one with \( f = 180 \) MHz, corresponding to the low- and high-frequency cases as described above. Signals are introduced into the plasma with a 4 cm diameter “antenna” centered at \((x,z) = (0,0)\).

V. RESULTS AND DISCUSSION

As discussed above, previous studies of ducted whistler wave propagation have primarily relied upon ray-tracing methods. For example, Stenzel17 presented results from both a laboratory experiment and a ray-tracing simulation to support the theoretical prediction of whistler propagation in density ducts. In a relatively uniform plasma such as that shown in the left panel of Figure 1, ray tracing is both valid and generally accurate; the wave’s behavior is expected to be that of a plane-wave which locally satisfies the dispersion relation. It should be noted, however, that the density decreases more rapidly near the edges of the domain than in the center, and the whistler might be better described as propagating in a broad duct rather than a uniform plasma. Even still, it should be expected that geometrical optics will provide an accurate model for whistler wave propagation in such a plasma. In the presence of a narrow density duct such as that seen in the right panel of Figure 1, a criterion for the validity of geometrical optics is violated (i.e., \( k_{\perp} \gg \nabla n/n \)). Thus, proper consideration of propagation in a narrow duct requires a full-wave solution to the EMHD equations. Our simulation results demonstrate that a plasma configuration which permits ray-tracing methods is also amenable to full-wave solution; however, we also find that in cases when ray-tracing would fail (as indicated by a violation of the
conditions for geometrical optics and a breakdown of the local dispersion relation) solutions can still be readily obtained from the full-wave method of solution discussed above.

A. High-frequency case ($\omega = 0.68\Omega_c$)

The density profiles used are shown in Figure 1. In both the non-duct and duct cases, a 180 MHz wave is introduced into the system. The absolute density and parallel wavelength of the driving signal are different for each case. A comparison of the experimental and simulated wave forms is shown in Figure 2.

1. Propagation in a plasma without a duct

Based upon the experimental data, a parallel wavelength of $\lambda_0 = 8$ cm is specified, resulting in a cutoff density of $n_0 = 8.23 \times 10^{10}$ cm$^{-3}$. The density at the center of the antenna is taken to be $n = 7.25 \times 10^{10}$ cm$^{-3}$. For the given plasma parameters, the dispersion relation (4) predicts a perpendicular wavelength of 18 cm, the width of the data taking region. Although a negative excursion would be expected near the edges (a centered half-wavelength would extend only to $\pm 4.5$ cm), the transverse variation of plasma density ($\lambda_0 \delta_c \propto n^{-1/2}$) and the presence of a nearby conducting boundary are likely responsible for producing the observed waveform. However, in the parallel direction where the finite size of the experiment is less important, we can use the motion of phase fronts (see Figure 3) to infer a parallel phase velocity of 0.05 c. This result consistent with the expected value for a plane wave with $f = 180$ MHz and $\lambda_1 = 8$ cm.

In Figure 2, it can be seen that the characteristics of the experimental and simulated waves are in good agreement, except near the antenna and the boundaries. Most notably, the experimental wave fronts exhibit slight distortions when compared to the simulated wave fronts. These distortions may arise from the interaction of the wave with its reflection off the machine walls, as well as minor variations in plasma density, which occur between runs (this will discussed in more detail below).

2. Propagation in a plasma with a duct

A parallel wavelength of $\lambda_1 = 14$ cm is specified ($n_0 = 2.7 \times 10^{10}$ cm$^{-3}$) and the plasma density at the center of the antenna is set to $1.6 \times 10^{10}$ cm$^{-3}$. The dispersion relationship (4) predicts a perpendicular wavelength of 19 cm for these parameters. However, as can be seen in Figure 2, the scale sizes of the ducted whistler differ considerably from the predicted values. This indicates the breakdown of the local dispersion relationship, an indication that the assumptions of geometrical optics are violated in this region. Instead of satisfying a simple local dispersion relationship, the waves in such a plasma satisfy a boundary eigenvalue problem.16 As a consequence,

![Figure 2](https://example.com/fig2.png)

**FIG. 2.** Comparison of experimental and simulated high-frequency whistler wave profiles. Top left: experimental $B_x$ wave field, uniform plasma; top right: experimental $B_y$ wave field, plasma with duct; bottom left: simulated $B_x$ wave field, plasma without duct; bottom right: simulated $B_y$ wave field, plasma with duct. Note that the experimental data have been normalized in order to put it on the same scale as the simulation data. This is justifiable because the wave is in a linear regime and its qualitative aspects—such as wavelength—should be independent of field amplitude.
the properties of the ducted wave are function not only of its source but also the structure of the plasma through which it propagates. We will not undertake to analyze the particular eigenstructure of this plasma, but will simply note that the perpendicular scale size of the ducted wave signal is essentially the width of the region where the density is lower than the critical density $n_0$, the previously discussed condition for the ducting of high-frequency whistlers.

Other one notable difference between the non-duct and duct cases is the deformation of phase fronts in the latter, which evolve into an extended V-shape as they propagate away from the source region. This is due to the spatial

![Simulation of high-frequency whistler wave propagation](image1.png)

**FIG. 3.** Simulated profiles of high-frequency whistler wave propagation for density profiles in Figure 1. Left column: Propagation in a plasma without a density duct; right column: Propagation in a plasma with a density duct.

![Experimental density profiles with and without duct](image2.png)

**FIG. 4.** Experimental density profiles used in simulation of low-frequency whistlers. Left: uniform density profile. Right: duct density profile.
variation of density in the duct (note \(v_g, v/\gamma d/\Delta n/C_0 = 2\)), since the signal in the center of the duct has a higher velocity and tends to outrace that at the edges. Outside the duct, the wave fronts are less distorted due to the increased uniformity of the plasma, but this is primarily evident near the edges of the duct due to the evanescence of the wave in this region. We can note a good agreement between the size and character of the wave structures in the duct, both along and across the magnetic field. This agreement is demonstrated in Figure 2, although small inter-run variations of plasma density may have caused the experimental waveform to become “fuzzy” and thus not entirely representative of the actual underlying structure. Thus, although the enhancements seen in the top-right panel of Figure 2 may appear to be slightly larger than those in the bottom-right panel, they are likely more even more similar than the figure would indicate.

B. Low-frequency case \((\omega = 0.34\Omega_{ce})\)

The density profiles used are shown in Figure 4. In both the non-duct and duct cases, a 90 MHz wave is introduced into the system. As in the high-frequency case, the absolute density and parallel wavelength of the driving signal are specified individually for each profile. A comparison of the experimental and simulated wave forms is shown in Figure 5.

1. Propagation in a plasma without a duct

A parallel wavelength of 14 cm is specified and the plasma density at the center of the antenna is set to \(n = 1.25 \times 10^{11} \text{ cm}^{-3}\). The dispersion relationship permits two solutions for this density, with \(\lambda_+^p = 10 \text{ cm}\) and \(\lambda_+^s = 17 \text{ cm}\) \((\lambda_+^p = 2\pi/k_+^p\)). Based on the experimental data in Figure 5, the longer-wavelength solution \((\lambda_+^s)\) was selected. It can be seen in Figure 5 that there is good agreement between the scale sizes of the simulated and experimental waveforms. As in the high-frequency case, the simulated wave fronts are sharper and less distorted. In addition, the parallel phase velocity inferred from the motion of phase fronts in Figure 6 is 0.04 \(c\), consistent with the expected value for a plane wave with \(\lambda_+^p = 14 \text{ cm}\) and \(f = 90 \text{ MHz}\). Aside from notably smaller phase velocity (due to the increased density), there is little phenomenological difference between this case and the corresponding high-frequency whistler.

2. Propagation in a plasma with a duct

A parallel wavelength of 6 cm is specified, resulting in a cutoff density of \(n_0 = 6.76 \times 10^{11} \text{ cm}^{-3}\) and a critical density \(n_1 = 6.05 \times 10^{11} \text{ cm}^{-3}\). The density at the center of the antenna is set to \(n = 4.0 \times 10^{11} \text{ cm}^{-3}\), so the only real
solution to the dispersion relationship is $\lambda_1 = 1.7$ cm. As in the high-frequency case, the characteristics of the wave as predicted by the dispersion relationship do not agree with the wave profiles seen in Figure 5. The most interesting difference between this and the high-frequency ducting case is that the wave is not propagating in a density depletion, but in an enhancement. Although the plasma configuration shown in Figure 4 would seem amenable to depletion ducting, the wave is actually propagating in the density enhancement that bounds the expected duct. In contrast to the high-frequency case, where wave fronts formed into extended V-shapes, the wave fronts in this case propagate with much less distortion and with the mouth and point of the V reversed. These differences are due to the different types of duct; as this is an enhancement duct density decreases towards the edge of the duct and therefore the phase speed increases.

A close comparison of the experimental data in Figure 5 and the corresponding density profile in Figure 4 show a small offset between the center of the wave front (the lagging point of the V-shape) and the location of the density peak. However, as previously noted, the shape of the wave fronts is consistent with the center being located in the density peak; such a wave form is not consistent with a whistler propagating along a density gradient. This apparent contradiction is resolved if we suppose that the wave is propagating outside the plane of measurement. Although the image shown in Figure 4 is two-dimensional, the actual density profile is three-dimensional. Rather than being a depression between two peaks, it is actually more like a cylindrical shell. Moreover, this profile shows indication of three-dimensional structure—the magnitude of the left- and right-enhancements is different, and the duct appears to widen at one end in a non-symmetrical manner. If the wave were primarily confined to a density enhancement slightly off of the plane of measurement, then the curvature of the density profile would place the apparent path of the signals leaking from the duct in the observed location. A 3D probe drive capable of volumetric data would be able to fully resolve this discrepancy.

VI. CONCLUSIONS

Using numerical simulations, we have demonstrated that the propagation of whistler-mode waves in an inhomogeneous plasma may be modeled using the electron magnetohydrodynamics equations. Using a finite-difference approximation to the EMHD equations, we have modeled the propagation of both high- and low-frequency whistlers in uniform and duct configurations. In one case, there is a disagreement between the experimental data and the simulation results that is most
readily explained by wave ducting near to, but out of the plane of measurement. However, good agreement between our simulation results and experimental observations of both ducted and unducted whistlers is generally observed, indicating that our two-dimensional model captures most of the relevant physics despite reduced dimensionality.

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