Three-dimensional gyrokinetic simulation of the relaxation of a magnetized temperature filament

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An electromagnetic, 3D gyrokinetic particle code is used to study the relaxation of a magnetized electron temperature filament embedded in a large, uniform plasma of lower temperature. The study provides insight into the role played by unstable drift-Alfvén waves observed in a basic electron heat transport experiment [D. C. Pace et al., Phys. Plasmas 15, 122304 (2008)] in which anomalous cross-field transport has been documented. The simulation exhibits the early growth of temperature-gradient-driven, drift-Alfvén fluctuations that closely match the eigenmodes predicted by linear theory. At the onset of saturation, the unstable fluctuations display a spiral spatial pattern, similar to that observed in the laboratory, which causes the rearrangement of the temperature profile. After saturation of the linear instability, the system exhibits a markedly different behavior depending on the inclusion in the computation of modes without variation along the magnetic field, i.e., \( k_z = 0 \). In their absence, the initial filament evolves into a broadened temperature profile, self-consistent with undamped, finite amplitude drift-Alfvén waves. But the inclusion of \( k_z = 0 \) modes causes the destruction of the filament and damping of the drift-Alfvén modes leading to a final state consisting of undamped convective cells and multiple, smaller-scale filaments.

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I. INTRODUCTION

Electron heat transport across a magnetic field remains one of the active research topics within the plasma community, primarily because of its relevance to achieving controlled fusion. Steep cross-field pressure gradients in magnetized plasmas can lead to the spontaneous growth in temperature, density, and magnetic fluctuations once a certain threshold gradient is exceeded. These fluctuations give rise to complex heat transport processes that result in energy losses exceeding predictions based on classical transport due to Coulomb collisions.

Because of the worldwide interest in magnetic confinement fusion, the majority of the experimental and modeling studies of electron heat transport have addressed toroidal confinement devices (Refs. 2 and 3 and references therein). But due to the complex magnetic topology, multiple heating channels, and boundary conditions associated with the operation of such devices, the description of the transport dynamics in such an environment poses a formidable challenge.

To simplify the study of electron heat transport, a series of basic experiments have been performed over the past decade in the Large Plasma Device (LAPD) operated by the Basic Plasma Science Facility (BaPSF) at the University of California, Los Angeles (UCLA). Since the details of the experimental arrangement and the major findings have been previously published, only a brief description is given to set the background for the present computer simulation study. The generic experiment uses a small (3 mm diameter), single-crystal LaB\(_6\) cathode to inject a low-voltage electron beam into a strongly magnetized (1 kG), cold, afterglow-plasma. The low-voltage beam acts as an ideal heat source that produces a long (~8 m), narrow (~5 mm in radius) temperature filament that is well separated from the walls of the machine. The existence of a transition from a regime of classical transport to one of anomalous transport has been established through detailed measurements. During the period of classical transport, drift-Alfvén waves grow linearly, driven by the temperature gradient.

In an effort to elucidate the morphology of the structures that develop nonlinearly in the temperature filament, Shi et al. studied a simplified convection-diffusion model. It was based on postulating the existence of two, large amplitude drift-waves whose spatial pattern was suggested by the linear mode analysis. The highlight of the study was the formation of extended spatio-temporal structures that locally resulted in Lorentzian-shaped temporal pulses of positive and negative polarities in different regions of the temperature profile.

The present study utilizes an electromagnetic, 3D gyrokinetic particle simulation code to simulate the self-consistent, spatio-temporal relaxation of an initially prescribed temperature filament whose parameters closely match those in the LAPD heat-transport experiments. The principal goals of the present study are: (1) to utilize the diagnostic capabilities of the computer code to shed information on details of the dynamics that laboratory techniques are not able to probe; (2) to test the effectiveness of the gyrokinetic methodologies in simulating a transport experiment having minimal complexities; and (3) to better elucidate the nonlinear physics retained in the gyrokinetic formalism.

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The simulation is found to reproduce the predicted linear growth rate and frequency of the linearly unstable drift-Alfvén waves driven by the temperature gradient. The radial eigenfunctions display a spiral structure consistent with those observed in the experiment. The growing eigenmodes cause a rearrangement of the radial temperature gradient that modifies the radial structure of the modes and results in their saturation. But the behavior of the system beyond this saturation stage is found to depend crucially on whether or not modes without variation along the confinement field (i.e., \( k_z = 0 \)) are retained in the computation. In the absence of \( k_z = 0 \) modes, the initial temperature profile broadens, but is not destroyed. The constraints imposed by the conservation laws inherent in the gyrokinetic methodology result in a final state in which the component of the vector potential, \( A_z \), is not included. This implies that shear Alfvén waves coexist with the chaotic motion of the plasma particles. The parallel electron distribution function displays a flattening in the neighborhood of the phase velocity of the unstable modes. When \( k_z = 0 \) modes are retained in the computation, the initial temperature profile is destroyed, the amplitude of the Alfvén waves is reduced to a low level, and a final state consisting of undamped convective cells, and multiple temperature filaments is attained.

The outline of the paper is as follows. Section II describes the gyrokinetic simulation model and the initial setup of the temperature filament used in the study. Section III provides an assessment of the growth rate and frequency of the drift-Alfvén waves driven unstable by the temperature gradient. Section IV presents the simulation results obtained when \( k_z = 0 \) modes are included. Section V illustrates the relaxation in the absence of \( k_z = 0 \) modes. Section VI discusses the various findings and Sec. VII provides conclusions.

II. SIMULATION MODEL AND SETUP

The simulation model used in this study is based on the gyrokinetic Vlasov-Poisson-Ampere system for a low-beta plasma. The equations governing particle motion arise from the characteristics of the gyrokinetic-Vlasov equation. The self-consistent evolution requires the solution of a gyrokinetic-Poisson equation to obtain the perpendicular and parallel electric fields; the magnetic field perturbations arising from the parallel particle currents are computed via Ampere’s law. The canonical momentum formulation is implemented with an electrostatic potential \( \phi \), and a parallel electric field \( \mathbf{E} \), is not included. This implies that shear Alfvén waves are retained, but the compressional Alfvén waves are not, as appropriate for a low beta plasma and the experimental conditions of interest to this study. The detailed equations of motion and field equations for the full-F kinetic model are described in Ref. 20.

The total energy invariant of the closed system under consideration,

\[
U = \sum \left( \frac{m_s v^2_{||}^2}{2} + \mu_s B \right) + \int \frac{dV}{8\pi} \left( |\mathbf{E}|^2 + |\mathbf{B}|^2 \right),
\]

consists of the volume integral over the computational box of the energy densities of the self-consistent electric and magnetic fields \((\mathbf{E}, \mathbf{B})\), and the sum of the energy of each jth gyrokinetic particle, having conserved magnetic moment \( \mu_s \) and mass \( m_s \) for both species (electrons and ions). Equation (1) is evaluated as the system evolves and serves as a measure of code fidelity. The total energy is found to vary by less than 0.01% for the results reported here. Also, the field energy and particle energy are monitored separately, which allows for the evaluation of energy exchange between each particle species and the field fluctuations. The self-consistent motion of both ion and electron species is followed; ions are treated as gyrokinetic particles while electrons are treated as drift-kinetic (i.e., electron gyroradius effects are not included). The simulation box consists of periodic boundaries along the z-direction, which is taken as the direction of the confinement magnetic field, \( B_0 \). The code can be run with the axially uniform modes (i.e., \( k_z = 0 \)) included or suppressed. Perfectly conducting walls are set at a finite distance along the x and y directions, transverse to the confinement field. The plasma particles are specularly reflected when they reach these walls and are periodically re-introduced in the z-direction when they complete an axial transit. The core of the numerical code has previously been used to investigate collisionless tearing instabilities and internal kink modes.

In diagnosing the results of the simulation, various physical quantities of experimental relevance are generated from the position, \( \mathbf{r}_j(t) \), and velocity, \( \mathbf{v}_j(t) \), of each jth gyroparticle at time \( t \). An interpolation function \( I_s(\mathbf{r} - \mathbf{r}_j(t)) \) \( \propto (\Delta V)^{-1} \) is used to assign a count of the particles of species “s” found within a computational cell of 3D volume \( \Delta V = \Delta x \Delta y \Delta z \) and whose center is located at the computational grid point associated with physical position \( \mathbf{r} \). For the results reported here \( \Delta x = \Delta y = 0.0625c/\omega_{pe}, \Delta z = 69.5c/\omega_{pe} \) with 256 \( \times \) 256 \( \times \) 16 cells and 16 particles per cell.

The density of particle species s is obtained from the expression

\[
n_s(\mathbf{r}, t) = \sum_j I_s(\mathbf{r} - \mathbf{r}_j(t)),
\]

while the effective scaled “temperature” is found from

\[
T_e(\mathbf{r}, t) = \frac{\sum_j \left( v_{||j}(t) - \langle v_{||j}(\mathbf{r}, t) \rangle \right)^2 + \mu_s B}{m_s} I_s(\mathbf{r} - \mathbf{r}_j(t)) / 3v_{th}^2 \sum_j I_s(\mathbf{r} - \mathbf{r}_j(t)),
\]

where \( T_{e0} \) is the initial peak temperature of species s with corresponding thermal velocity \( v_{th} \). The average parallel velocity that is subtracted in obtaining the effective temperature is

\[
\langle v_{||j}(\mathbf{r}, t) \rangle = \frac{\sum_j v_{||j}(t) I_s(\mathbf{r} - \mathbf{r}_j(t))}{\sum_j I_s(\mathbf{r} - \mathbf{r}_j(t))}.
\]

The actual numerical calculations are performed on a Cartesian grid \((x, y, z)\), but, because the structure of interest, i.e., the electron temperature filament, is a cylinder, the...
computational results are interpolated onto a cylindrical mesh \((r, \theta, z)\) in order to elucidate the physical processes, e.g., the wave structures are described as having an azimuthal mode number “m” and an eigenfunction that varies with radial distance from the center of the initial filament. In figures presented in Sec. IV, various displays are made of the magnitude of the electrostatic potential of mode-m, \(|\Phi_{nm}|\), for specified values of \((r, z)\) at time t. This quantity is related to the calculated electrostatic potential \(\phi\) according to

\[
\phi(r, z, t) = \sum_n \sum_m \phi_{nm}(r, t) \exp \left[ i(m\theta + 2\pi nz/L) \right],
\]

where \(L\) is the axial length of the computational box.

The simulation is initialized with a spatially uniform density of ions and electrons \((n_i(r, t = 0) = n_0)\) but the electron temperature is non-uniform; it consists of a narrow filament of elevated temperature embedded in a cold background plasma, as illustrated in Fig. 1. The initial radial variation of the electron temperature is chosen to closely match the quiescent (classical transport phase) experimental profiles and is given by

\[
T_e(r) = T_b + (T_o - T_b) e^{-r^2/r_s^2},
\]

with the temperature of the cold background electrons, \(T_b = 0.25\) eV, the peak temperature of the filament, \(T_o = 5\) eV, and the filament width, \(r_s = 0.3\) cm, in physical units (cgs). The initial radial variation in electron temperature is implemented in the code at the distribution function level by applying Gaussian random loading with a thermal velocity width consistent with the electron temperature profile given by Eq. (7). Gaussian random loading is also applied to the parallel and perpendicular velocities. The ion temperature is taken to be spatially uniform with value, \(T_i = 0.1\) eV. It is found that over the length of time considered in this study, the changes in ion temperature are comparable to the level of numerical fluctuations, i.e., ion heating is not a significant effect in the relaxation process.

In the experiments, the axial length of the temperature filament is about 8 m and the transverse dimension is about 5 mm, whereas the surrounding plasma is 18 m long and 75 cm in diameter. The scaled simulation parameters are chosen to match the scaled parameters corresponding to the experiment and these are obtained from the typical experimental values of electron density \((n_e = 1.4 \times 10^{12} \text{ cm}^{-3})\), magnetic field \((B_0 = 1 \text{ kG})\), and temperatures \((T_e = 5\) eV, \(T_i = 0.1\) eV). A fundamental scale-length obtained from these parameters, and used for normalization in the simulation, is the electron skin depth, \(c/\omega_{pe} = 0.45\) cm. The simulation domain in the x-y plane across the magnetic field is chosen to be large enough to minimize the influence of the boundaries on the results. From several test runs, it has been found that the smallest transverse system size that can be used without significant boundary influence on the drift-Alfvén eigenmodes is \(8c/\omega_{pe} \times 8c/\omega_{pe}\). The results shown in this manuscript are obtained with a transverse system size \(16c/\omega_{pe} \times 16c/\omega_{pe}\). From the physical parameters the ion sound gyroradius is \(\rho_s = c_s\Omega_i = 1.02c/\omega_{pe}\), where \(c_s\) is the ion sound speed using the central electron temperature of the filament and in terms of the ion gyroradius the transverse system size is \(113.3\rho_i \times 113.3\rho_i\). The axial periodicity of the simulation box corresponds to \(L = 5\) m in the experiment. For these parameters, the ratio of the total kinetic energy of the initial hot filament to the total kinetic energy of the cold plasma contained within the simulation box is \(4 \times 10^{-2}\).

It should be noted that in this simulation the heating source (an electron beam), the electron-ion collisions, and the boundary losses (radial and axial), which are all present

![FIG. 1. Schematic of the 3D simulation geometry and temperature filament embedded in a cold background plasma. Axial boundary conditions are periodic. Transverse walls are perfectly conducting and ideal particle reflectors. The x-coordinate in the inset is scaled to the electron skin-depth; only 1/4 of the computational box is shown. Inset arrows (red) refer to virtual probe locations. The temperature is scaled to the peak temperature of the filament.](image-url)
in the experiment, are not included. Therefore, the present study addresses only the collisionless relaxation aspects of the experiment; in the simulation, the initial temperature gradient is relaxed solely by the action of the self-consistent electric and magnetic field fluctuations. The simulation includes resonant particle transport, a process that is absent in the previous diffusion-convection study in which only the fluid \( \text{Ex} \times \text{B} \) drifts are modeled. In the gyrokinetic simulation, each particle has a conserved magnetic moment. This implies that the total perpendicular energy in the system, regardless of the complex dynamical processes, is strictly conserved because in this study the magnetic field is uniform (the contribution from the perpendicular kinetic energy term, \( \mu_i \), in Eq. (1) remains constant in time). This constraint has consequences for the energetics of the relaxation sampled, since it is only the parallel wave-particle interactions that account for the exchange of energy with the fluctuations. For the conditions investigated, only 1.3% of the total system energy is available (i.e., the parallel energy of the filament) to excite fluctuations and heat cold particles. Of course, cross-field transport occurs simultaneously by \( \text{E} \times \text{B} \) convection, as in the fluid model, but without exchange of energy.

III. TEMPERATURE GRADIENT MODES

It is useful to first consider a simple, local description of the drift-Alfvén instability associated with a pure electron temperature gradient in order to identify the parameter dependencies. The relevant dispersion relation is given by Eq. (30) of Ref. 12,

\[
k^2 \beta^2 + \left( \frac{\omega^2}{k_V^2} - 1 \right) \left[ \frac{\omega_i}{\omega} + \frac{1}{2} Z^2 \left( \frac{\omega_e}{\omega} \right) \right] = 0, \tag{8}
\]

where the local electron diamagnetic drift frequency is \( \omega_d = k_0 \sqrt{2} \mu_i / \Omega_e \), with \( \Omega_e \) the electron gyrofrequency, \( \omega_e / \omega = \omega / (\sqrt{2} k_V V_A) \), \( L_T = |\nabla T / T|^{-1} \) is the temperature scale length, \( k_0 = m_i / r \) is the local azimuthal wave number, and \( \rho_i = c_s / \Omega_i \) is the ion sound gyroradius, defined as the ratio of the ion sound speed, \( c_i \), to the ion cyclotron frequency, \( \Omega_i \). The local electron thermal velocity is \( V_T \) and the Alfvén speed is \( V_A \), while \( Z^2 \) corresponds to the derivative, with respect to argument, of the standard plasma dispersion function. In Eq. (8), the terms inside the left bracket describe the zero electron mass (MHD) shear Alfvén wave branch, while the terms inside the right bracket describe the collisionless, drift-wave branch. For small transverse scales (i.e., large \( k_0 \)), as is appropriate for this study, the two branches are coupled and result in a collective mode known as the drift-Alfvén wave, which can become unstable for certain values of the azimuthal mode number, \( m \), and axial wave number, \( k_z \). Because of this coupling, the modes carry electrostatic as well as magnetic energy as is illustrated in Sec. IV.

The accurate calculation of the linear growth rates and frequencies requires the solution of coupled differential equations for the electric and magnetic potentials to determine the complex eigenfrequencies and the associated eigenfunctions that satisfy the proper boundary conditions. Such an analysis has been performed for the temperature filament considered here and the details have been reported in Ref. 12. Figure 2 shows a comparison of those results to modes that are found to grow exponentially during the early, linear stage of the simulation. The dependence on azimuthal mode number of the frequencies and growth rates, scaled to the ion gyrofrequency, is displayed. Good agreement is obtained between the gyrokinetic simulation and the predictions of linear theory. Some noteworthy features are: the range of excited azimuthal mode numbers is \( m \sim 1–6 \), the frequency of oscillations is \( \Re \omega / k_z v_{pe} = 0.4 \), where \( v_{pe} \) corresponds to the electron thermal velocity at the peak temperature of the initial filament. For the parameters of this study, the value of the Alfvén speed, \( V_A \), is approximately equal to \( v_{pe} \). Although not shown here, the shape of the radial eigenfunctions found in the simulation also agree with those predicted theoretically. It should be mentioned here that in order to obtain the close agreement shown in Fig. 2, low initial noise levels using quiet-start particle loading procedures (bit-reversed quasi-random number sequences in velocity space) and perturbing single azimuthal and axial mode numbers are employed.

IV. SIMULATION RESULTS

A. Evolution of fluctuations

An overview of the temporal evolution of the fluctuations spontaneously excited by the electron temperature filament can be obtained from the results displayed in Fig. 3. It should be emphasized that the results presented in this section are obtained with the \( k_z = 0 \) modes included in the computation. A comparison to results with the \( k_z = 0 \) modes suppressed is given in Sec. V. The top panel, Fig. 3(a), illustrates the fractional changes in the various components of the energy stored in the simulation box, scaled to the initial
transformation can be identified from the behavior of the magnetic energy (bottom green curve in Fig. 3(b)). It is seen that through the linear growth and saturation stages ($\Omega t < 400$) the behavior of the magnetic energy essentially follows that of the electrostatic energy, as expected for a drift-Alfvén mode. But for $\Omega t > 400$ this component of the energy undergoes a steady damping and eventually achieves an asymptotic, relatively small level. This decay in the magnetic energy signals that the energy stored in the drift-Alfvén modes by the linear instability is re-shuffled. From panel 3(a) it is seen that some of this energy is returned to the particles (bottom green curve), but most of it is transformed into electrostatic energy, as indicated by the nearly constant level of the total field energy (top blue curve in 3(a)) for $\Omega t > 600$.

Figure 4 displays the time evolution, in log-linear format, of the electrostatic potential mode-amplitude given by Eq. (6), scaled to the peak of the electron temperature, $e |\Phi_m(r, t)| / T_0$, and radially averaged over the interval $r = 0 - 4c / \omega_{pe}$. Shown is the behavior of azimuthal mode numbers $m = 1 - 5$. In the top panel, Fig. 4(a), the contribution from modes with axial mode number $n=0$ is not included, i.e., the summation in Eq. (6) does not contain $k_z=0$ modes (but these modes are in the simulation and are an integral part of the dynamics shown in (a)). The bottom panel, Fig. 4(b), shows explicitly the behavior of the $k_z=0$ modes, i.e., modes with $n \neq 0$ are not included in the...
n-summation in Eq. (6). Figure 4(a) shows that for $\Omega t < 200$ the $k_z \neq 0$ modes grow exponentially in time, as expected from linear theory. It has been checked, but not shown here, that modes in both axially traveling directions (i.e., $n > 0$ and $n < 0$) are unstable. The $k_z \neq 0$ modes reach saturation at $\Omega t = 200$ and all experience damping for $\Omega t > 300$, with mode $m = 1$ becoming the dominant mode in the asymptotic state. Figure 4(b) shows that, from the earliest times, all the $k_z = 0$ modes exhibit exponential growth while the $k_z \neq 0$ modes still have relatively small amplitude. Also, their growth rates are comparable to those of the linearly unstable $k_z \neq 0$ modes. As shown in Sec. V, in the absence of the $k_z \neq 0$ modes, the $k_z = 0$ modes experience very small direct amplification from the zero-order electron temperature gradient. The implication of these observations is that the origin of the $k_z = 0$ modes is not a parametric or a modulational instability, but rather their direct excitation is caused by the bilinear beating of the linearly unstable $k_z \neq 0$ modes. This behavior is analogous to the process identified by Cheng and Okuda in an electrostatic simulation of drift waves driven unstable by a density gradient.

Figure 5 displays, in log-linear format, the time evolution of the total energy (scaled to $U_0$) associated with $k_z = 0$ structures. The curve (blue) achieving the largest value is the electrostatic field energy and the smaller curve (green) is the magnetic field energy. It is seen that for $\Omega t < 300$ the $k_z = 0$ electrostatic energy increases steadily; it is during this interval that drift-Alfvén modes are linearly unstable. For $\Omega t > 300$ the energy that has been transferred from the drift-Alfvén modes to the $k_z = 0$ structures remains unchanged. From the behavior of the magnetic energy, it is seen that relatively little energy is transferred to field-aligned current filaments, although some excitation of such structures does take place. This comparison indicates that the $k_z = 0$ structures are essentially electrostatic convective cells that exhibit no damping after being excited. They contain about 1% of the parallel kinetic energy of the initial filament.

B. Spatial patterns

Figure 6 shows 2D color contours, across the confinement magnetic field, of the instantaneous electrostatic potential $\phi(x, y, z, t)$, in the left panels, and of the magnitude of the fluctuating magnetic field $|B_x(x, y, t)|$, in the right panels. It presents a snapshot of these quantities within a transverse plane located at the axial midpoint of the computational box. The top two panels, Figs. 6(a) and 6(b), are at time $\Omega t = 180$ during the linear growth stage, while the bottom two panels, Figs. 6(c) and 6(d), are at the saturation phase, $\Omega t = 380$. Figure 6(c) exhibits an extended $m = 1$ spiral structure (rotating in the electron diamagnetic direction).
quite similar to the structures observed in the experiments, e.g., Fig. 4 of Ref. 7 and Fig. 5 of Ref. 10. It is seen that the electrostatic component of the eigenmodes has broader radial extent than the magnetic counterpart. A closer inspection of the small-r region of the spiral in Fig. 6(c) shows that small-scale concentrations of electrostatic potential are embedded within the more extensive eigenmode. These local enhancements correspond to the convective cells.

Figure 7 displays 2D color contours of the instantaneous electrostatic potential $\phi(x, y, z, t)$, in the left panels, and of the $y$-component $B_y(x, y, z, t)$ (azimuthal component at this position) of the fluctuating magnetic field, in the right panels. It presents a snapshot of these quantities within a plane that illustrates the axial structure corresponding to a cut along a fixed azimuthal angle $\theta = 0$. It should be noted that the axial extent displayed (z-direction) is approximately a factor of 63 larger than the transverse extent (x-direction), as is required to capture the filamentary nature of the fluctuations. The top two panels, Figs. 7(a) and 7(b), correspond to a time $\Omega_t = 180$ during the linear growth stage, while the bottom two panels, Figs. 7(c) and 7(d), are at the saturation phase, $\Omega_t = 380$. They complement the transverse view shown in Fig. 6. From the top panels, Figs. 7(a) and 7(b), it is seen that the linearly unstable modes have parallel wavelengths comparable to the length of the temperature filament. In the saturation phase, Figs. 7(c) and 7(d), both the electrostatic potential and the magnetic field develop significant cross-field structures; they result from the modifications of the temperature profile. Also, visible elongations in Fig. 7(c) indicate the development of significant contributions to the total electrostatic potential from $k_z = 0$ modes.

C. Profile relaxation

Figure 8 provides a visual summary of the four stages associated with the relaxation of the temperature filament. Shown are 2D color contours of the electron temperature on a plane $(x, y)$ transverse to the confinement field, at an axial location (z-position) midway in the computational box. The top-left panel, Fig. 8(a), shows the early stage during which the drift-Alfvén modes grow exponentially. The filament remains essentially circular but has small perturbations. The top-right panel, Fig. 8(b), corresponds to the stage when the linear instability has saturated and modest expansion of the profile takes place. Up to this point, profile expansion occurs due to $E \times B$ flows driven by the large amplitude drift-Alfvén modes. The bottom-left panel, Fig. 8(c), hints at a post-saturation stage in which the profile is undergoing a fission process in which the large initial filament breaks into smaller filaments. The fission process is due to the increasing dominance of the $k_z = 0$ modes. The bottom-right panel, Fig. 8(d), illustrates that the fission event results in an asymptotic state consisting of multiple filaments located far from the initial filament and having smaller scale. The choice of computational grid size and number of particles per cell is based on the parameters of the initial temperature filament. While the values used are adequate to resolve the linear growth and saturation, they may not be sufficient to resolve the finer spatial...
scales generated at later times. A detailed quantitative study of the dynamics of the asymptotic state requires a separate, high-resolution simulation.

Figure 9 exhibits the details of the radial relaxation of the initial temperature filament caused by the fluctuations. The quantity displayed is the effective electron temperature defined in Eq. (3), but now averaged over the axial direction, $z$-axis, and azimuthal angle $\theta$, i.e., the radial profile, $T_e(r, t)$, scaled to the initial peak temperature $T_0$. The top panel, Fig. 9(a), displays the radial dependence of the profile for different times spanning the interval $0 \leq \Omega t \leq 905$, covered in Fig. 8. It is seen that, by the time the fluctuations are well into the saturation stage, $\Omega t = 380$, corresponding to the green curve, the peak temperature has dropped by about 40% and the maximum temperature gradient is relaxed by about a factor of 2. By this time, the spreading of the temperature extends to about twice the initial width of the filament. A drastic collapse of the profile occurs beyond this saturation stage to the saturation stage before the temperature profile relaxes, the eigenmodes stretch outward and develop more radial oscillations (larger perpendicular wave number), as suggested by the contour displays of Figs. 7(c) and 7(d).

The bottom panel, Fig. 9(b), documents the temporal variation of the electron temperature at various radial positions indicated by the arrows shown at the bottom of the frame in Fig. 9(a); they span a region extending near the center of the initial filament to a location well into the cold background plasma. It is seen that the interior of the filament undergoes a gentle relaxation during the early saturation phase and it is followed by a rapid collapse with a propagating pulse behavior for times $\Omega t > 300$. The speed of propagation of the heat pulse is approximately $V_{\text{pulse}} \approx 0.0075 (c/\alpha_{pe}) \Omega$, which in physical units becomes $V_{\text{pulse}} \approx 8.1 \times 10^5 \text{ cm/s}$ for $c/\alpha_{pe} = 0.45 \text{ cm}$ and $\Omega = 2.4 \times 10^5 \text{ s}^{-1}$, which is a factor of 10 smaller than the sound speed of the cold plasma.

Comparing the observed profile relaxation to Figs. 4(a) and 4(b) indicates that the first stage, consisting of a relatively gentle expansion of the temperature profile, is associated with the saturation of the linearly unstable drift-Alfvén waves. But the drastic collapse of the profile after this saturation is caused by the dominance of fluctuations having $k_z = 0$ structure in the interval $\Omega t > 400$.

Figure 10 illustrates the radial rearrangement of the eigenfunctions of the drift-Alfvén modes, from the linear stage to the saturation stage before the temperature profile collapses. The left panels, Figs. 10(a) and 10(c), correspond to the scaled magnitude of the electrostatic potential for mode-$m$ given by Eq. (6), i.e., $|\Phi_m(r, t)|$, and the right panels, Figs. 10(b) and 10(d), to the equivalent quantity for the magnitude of the perpendicular component of the magnetic field $|B_\perp(r, t)|$. Within each panel, modes $m = 1, 2, 3$ are shown. Superposed in each panel is the corresponding electron temperature profile $T_e(r, t)$ (red curve), whose numerical value is shown on the right side. The top panels, Figs. 10(a) and 10(b), are for the linear stage $\Omega t = 180$, and the bottom panels, Figs. 10(c) and 10(d), are for the saturated stage $\Omega t = 905$. It is seen that in the linear stage the magnetic eigenfunctions are more localized than the electrostatic eigenfunctions, consistent with Fig. 6. As the temperature profile relaxes, the eigenmodes stretch outward and develop more radial oscillations (larger perpendicular wave number), as suggested by the contour displays of Figs. 7(c) and 7(d).

FIG. 8. Color contour displays of electron temperature across confinement magnetic field at fixed times: (a) $\Omega t = 180$, (b) $\Omega t = 380$, (c) $\Omega t = 530$, and (d) $\Omega t = 905$. 

FIG. 9. 

FIG. 10.
Zonal flows arise from the subset of the $k_z = 0$ modes having no azimuthal dependence, i.e., $m = 0$. By themselves they do not cause radial transport of the filament, but can alter the stability properties of the drift-Alfvén modes and modify their radial eigenfunctions. The evolution of the magnitude of the electrostatic potential driving zonal flows is shown in Fig. 11. The quantity displayed is the modal amplitude defined in Eq. (5), $|\phi_{00}(r, t)|$, for $m = 0$, $n = 0$, scaled to the peak initial temperature of the filament. The sequence of panels (a)–(d) shows the spatio-temporal evolution over the interval $180 \leq \Omega_t \leq 905$, ranging from the linear instability stage to the later stage when the filament has been destroyed. In each panel, the corresponding electron temperature profile is superposed (red curve) and its scaled value is indicated on the right axis. It is seen that the zonal flows are present during the linear growth stage, i.e., they are driven by the unstable drift-Alfvén modes, as are the other $(n = 0, m \neq 0)$ modes shown in Fig. 4(b). As the temperature profile expands the zonal flows adjust their radial pattern, and after the collapse stage they track the radially moving heat pulse, as seen in Fig. 11(c) for $\Omega_t = 530$. It is noteworthy that at the late times when the temperature filament has been destroyed, the zonal flows survive, as seen in Fig. 11(d) at $\Omega_t = 905$, and extend far from the initial region occupied by the temperature filament.

D. Probe signals

Motivated by the use of local probes in laboratory experiments to measure the temporal evolution of fluctuating quantities, the results of the simulation are sampled at four different positions. The “simulation probes” are equally spaced azimuthally, at the axial (z-direction) midpoint of the computational box, and at a fixed radial location $r/(c/\omega_{pe}) = 1$, indicated by the arrows in the inset in Fig. 1. Figure 12 displays the temporal behavior of the electrostatic potential (red curve, scale on the left) and the azimuthal component of the fluctuating electrostatic potential (blue curve, scale on the right) for the azimuthal mode numbers $m = 1–3$, at fixed times during (a) and (b) linear phase ($\Omega_t = 180$), and (c) and (d) saturation phase ($\Omega_t = 380$). Superposed red curve in each panel is the temperature profile whose value is given by the scale on the right-side.
magnetic field (blue curve, scale on the right) for the probe located at azimuthal angle $\theta = 0$. The temporal axis is shown in dual scale; scaled time $\Omega t$, and actual time relevant to the experiments. It is seen that the signals exhibit clear oscillations associated with the drift-Alfvén modes, but their amplitude and phase change as the profile relaxes.

Figure 13 displays the ensemble average (over the four probes) of the frequency power spectrum of the fluctuations in electrostatic potential and in the azimuthal component of the magnetic field. The top panel uses a log-log format and the bottom panel a log-linear format. Again a dual presentation is given in terms of the frequency, $f$, scaled to the ion cyclotron frequency, $f_{ci}$, and also the frequency in kHz for direct comparison to the experiments. The predicted frequency of the most-unstable mode is indicated by the top arrows in both panels. It is seen that the spectrum of the potential displays a sharp peak at a frequency (34 kHz) corresponding to the theoretical prediction and also in close agreement with the value measured in the experiments.\textsuperscript{8,10} The magnetic fluctuations have a broader spectrum than the potential fluctuations, but both spectra have a common baseline that exhibits a power-law frequency dependence, i.e., as

![Figure 11](image1.png)

**FIG. 11.** Radial variation of the magnitude of the electrostatic potential mode of Eq. (5) for azimuthal mode number $m = 0$ and axial mode number $n = 0$ (zonal flow mode) at fixed times during: (a) linear phase ($\Omega t = 180$) and saturation phase (b) $\Omega t = 380$, (c) $\Omega t = 530$, and (d) $\Omega t = 905$. Superposed red curve in each panel is the temperature profile whose value is given by the scale on the right.

![Figure 12](image2.png)

**FIG. 12.** Time series of the scaled electrostatic potential and scaled azimuthal magnetic field (B-theta) taken at a fixed spatial position, $r = 1c/\theta_{pe}$, outside the filament.

![Figure 13](image3.png)

**FIG. 13.** Ensemble average frequency power spectrum of the electrostatic potential and azimuthal magnetic field, (a) log-log format and (b) log-linear format. The arrow labeled $f_{th}$ shows the value of the dominant mode frequency predicted by linear theory.
adding noise to a chaotic process has been investigated by Rosso et al.22 Adding noise to a chaotic process moves the location in the C-H plane towards the location of the pure noise signal along a path that roughly parallels the maximum complexity curve. The change in the C-H plane location depends upon the character of the noise and the ratio of the noise amplitude to that of the chaotic process (see Fig. 7 of Rosso et al.26 for details). To ameliorate the contribution of computational noise to the electron temperature signals, the signal from each probe location is averaged over four neighboring grid points to obtain a spatially averaged temperature that has a lower noise content. The C-H plane location of the temperature displayed in Fig. 14 is for the un-subsampled, spatially averaged temperature (i.e., d = 5, s = 1). It is found that all these signals fall in the region, within the C-H plane, associated with chaotic processes, i.e., above the fractional Brownian motion curve (labeled “fBm”) and having moderate entropy. The location of these simulation signals in the C-H plane is consistent with the results shown in Fig. 6 of Ref. 23 for the experimental observations, and also for the results of a chaotic advection model of the filament evolution.

E. Particle orbits

To explore how individual particles are affected by the growing fluctuations and how their motion eventually results in the relaxation of the temperature filament, the orbits of a select group of self-consistent particles, not test particles, are monitored. For this purpose a group of 10^6 electrons and 10^4 ions is chosen before the simulation is run; their velocity distribution is essentially Maxwellian. The initial positions lie inside a cylindrical region having radius 0.5c/\nu_p, from the center of the simulation domain and uniformly distributed along the axial direction. Figure 15 displays a sequence of instantaneous particle positions in the x-y plane (i.e., across the magnetic field), for the electron ensemble. The top left-panel, Fig. 15(a), shows a uniform disk corresponding to the starting positions. The top-right, Fig. 15(b), indicates that during the linear growth stage, \Omega t = 180, the disk broadens

![FIG. 14. C-H (complexity-entropy) plane display of fluctuations in electrostatic potential, azimuthal magnetic field, and electron temperature sampled by four different probes, equally spaced azimuthally, at radial position, r = 1c/\nu_p, as indicated in Fig. 1. All the signals are found to be in the region associated with chaotic dynamics.](image)

![FIG. 15. Positions across the confinement magnetic field (x-y plane) of 10^6 electrons that initially were located in the central region of the temperature filament. Times shown are: (a) \Omega t = 0, (b) \Omega t = 180, (c) \Omega t = 380, and (d) \Omega t = 800.](image)
and develops outstretched arms mirroring the structure of drift-Alfvén modes. The bottom left-panel, Fig. 15(c), during the saturation phase $\Omega_t = 380$, shows a broad spreading of the particles whose shape neither reflects the shape of the initial filament nor the structure of the drift-Alfvén modes. The bottom right-panel, Fig. 15(d), at $\Omega_t = 800$, after the collapse shown in Fig. 8(b) occurs, shows several well-separated patches that are uniformly populated. They give the impression of a system undergoing internal mixing but subjected to rapid fission into fragments, akin to the splitting of a liquid into drops.

To quantify the spreading of the orbits, the magnitude of the 3D spatial separation between initially neighboring particles, $\Delta$, is calculated as a function of time and the ensemble average is constructed for both the electron and ion populations. To implement the procedure another group of code particles, consisting of 1600 electrons and 1600 ions, is chosen before the simulation is run (both groups have Maxwellian velocity distributions) with initial positions that lie inside a small, non-concentric cylindrical region having radius $0.1c/\omega_{pe}$ and uniformly distributed along the axial direction. The center of the cylinder is located at $r = 0.5c/\omega_{pe}$, near the point where the initial temperature gradient is largest. The resulting behavior is shown in Fig. 16 in log-linear format. It is seen that during the interval $\Omega_t \leq 400$, corresponding to the linear and saturation stages, the orbit separation grows exponentially in time, and at a common rate for electrons and ions which is comparable to the linear growth rate of the drift-Alfvén modes. This suggests a stage in which chaotic motion due to ExB flows driven by the drift-Alfvén modes results in a charge-neutral relaxation of the temperature profile, consistent with the mild profile change seen in Fig. 9(a) over this time interval. But Fig. 16 shows that for $\Omega_t > 400$, i.e., during the linear growth, and up to the saturation stage, the effect of the $k_z = 0$ modes is negligible. This implies that the moderate rearrangement of the temperature profile that takes place before the collapse occurs, as shown in Fig. 9(a), is caused by the chaotic motion induced by the drift-Alfvén modes.

**V. RELAXATION WITHOUT $k_z \neq 0$ MODES**

This section explores the intrinsic relaxation of the temperature filament caused by $k_z \neq 0$ modes and makes comparisons to the results presented in Sec. IV. Figure 17 illustrates the widely different asymptotic behavior (i.e., $\Omega_t > 600$) exhibited by the electrostatic energy, top panel Fig. 17(a), and magnetic energy, bottom panel Fig. 17(b), of the $k_z \neq 0$ modes. In each of these panels, the top curves (green) are obtained with the $k_z = 0$ modes suppressed in the code, and the bottom curves (blue) with these modes included. It is evident that without convective cells the drift-Alfvén modes achieve a larger amplitude at saturation and thereafter remain undamped. It is significant that for $\Omega_t < 200$, i.e., during the linear growth, and up to the saturation stage, the effect of the $k_z = 0$ modes is negligible. This implies that the moderate rearrangement of the temperature profile that takes place before the collapse occurs, as shown in Fig. 9(a), is caused by the chaotic motion induced by the drift-Alfvén modes.

Figure 18 illustrates the orbit spreading experienced by the select group of electrons discussed in Sec. IV D, as shown in Fig. 15, but now in the absence of $k_z = 0$ modes. Up to $\Omega_t = 180$, the behavior is essentially the same, but at a slower rate than the electrons, suggesting that charge separation occurs. It is during this interval that convective cells ($k_z = 0$) dominate.

![FIG. 16. Time evolution, in log-linear format, of ensemble-average of the square of the separation $(\Delta^2)$ between neighboring tagged particles (electrons and ions). Functional fits of the form, $\exp(x\Omega_t)$, are indicated by the straight lines and the value of $x$ is shown.](image1)

![FIG. 17. Time evolution of field energies, scaled to the total system energy $U_0$, with and without $k_z = 0$ modes included in the simulation: (a) electrostatic energy and (b) magnetic energy.](image2)
after saturation, i.e., $\Omega \tau > 380$, it is quite different. Now the constant amplitude drift-Alfvén modes cause a uniform spreading that results in the slow expansion of the temperature filament.

Figure 19 documents the gentle expansion of the temperature profile associated with the sampled orbits in Fig. 18. It is to be compared to the behavior shown in Fig. 9(a), where the convective cells are included.

In addition to the different radial relaxation that occurs in the absence of convective cells, there is also a significant difference in the modifications caused in the parallel electron distribution function, as is illustrated in Fig. 20. In both panels the dotted curve corresponds to the initial parallel electron distribution function of the $10^4$ tagged electrons in the center of the filament, and the continuous curve to the asymptotic value at $\Omega \tau = 800$. In each panel, the arrows indicate the Alfvén speed, $V_A$, and the phase velocity, $V_p$, of the most unstable mode. The top panel, Fig. 20(a), is the result obtained in the absence of $k_z = 0$ modes, and the bottom panel when they are included. It is seen that local flattening of the distribution function (in both axial directions) and major slowing down (cooling) take place when the drift-Alfvén modes act alone. But when the convective cells are present, only a minor rearrangement is seen.

To ascertain that the generation of the convective cells is linked to the growth of the drift-Alfvén modes, two idealized simulations are performed in which the $k_z \neq 0$ modes are suppressed. One is started with the same initial conditions leading to the results in Sec. IV, i.e., a narrow filament embedded in a cold plasma. The other considers a plasma of uniform electron temperature having a value equal to the peak temperature of the filament, i.e., $T_e = 5$ eV. The key results are summarized by Fig. 21 in which the top panel corresponds to the uniform, thermal run, and the bottom panel

![Figure 18](image1.png)

**FIG. 18.** Positions across the confinement magnetic field (x-y plane) of $10^4$ electrons that initially were located in the central region of the temperature filament. Times shown are: (a) $\Omega \tau = 0$, (b) $\Omega \tau = 180$, (c) $\Omega \tau = 380$, and (d) $\Omega \tau = 800$, for the case with $k_z = 0$ modes excluded from the simulation. To be compared with Fig. 15.

![Figure 19](image2.png)

**FIG. 19.** Relaxation of the electron temperature profile over the interval $0 \leq \Omega \tau \leq 905$ for simulation without $k_z = 0$ modes included. To be compared to Fig. 9(a).

![Figure 20](image3.png)

**FIG. 20.** Parallel velocity distribution function associated with the $10^4$ tagged electrons initially located at the center of the filament. Dotted curve is the initial Maxwellian ($\Omega \tau = 0$) and solid curve is the value in the asymptotic state ($\Omega \tau = 800$) for the cases: (a) without $k_z = 0$ mode included in the simulation and (b) with $k_z = 0$ mode included.
to the case with the temperature filament present. The curves in Fig. 21 are the potential mode-amplitudes given by Eq. (6), scaled to the peak of the electron temperature, \( e\Phi_m(r, t)/T_0 \), and radially averaged over the interval \( r = 0 - 4c/\Omega_{pe} \), as done in Fig. 4, but now the summation over the axial mode numbers has only the contribution from \( n = 0 \) (\( k_z = 0 \)). The range of azimuthal mode numbers \( m = 1-5 \) is shown. It is seen that in the uniform temperature plasma there is a finite level of convective-cell fluctuations. After an initial transient they settle to an essentially constant level which is two-orders of magnitude smaller than that shown in Fig. 4(b). The bottom panel, Fig. 21(b), shows that the temperature filament can cause the growth of convective cells, but at a relatively small growth rate (<0.005 \( \Omega_r^{-1} \)) that cannot result in the large amplitude fluctuations in Fig. 4(b), where the drift-Alfvén modes are active.

VI. DISCUSSION

As mentioned in the Introduction, this study aims to provide insight into the LAPD heat transport experiments, to assess the capabilities of the gyrokinetic model, and to elucidate related nonlinear processes. In this section, a discussion is presented of what has been learned in the pursuit of these goals.

Although it is evident from the outset that the simulation model does not contain important elements present in the LAPD experiments, such as Coulomb collisions, open axial boundaries, and a heating source, it is found that several features measured in the laboratory are well-reproduced. A central one is the linear instability of drift-Alfvén waves. The unstable modes display a radial eigenmode structure similar to that observed in the experiments, having both an electrostatic and a magnetic character, even at the relatively small value of the plasma-beta parameter considered. The frequency of the dominant mode in the simulation power spectrum is 34 kHz, which is to be compared to the value of 38 kHz measured in the early study by Burke et al. and of 30 kHz in the more recent experiment by Pace et al. In the saturation stage, the simulation shows that the cross-field fluctuations are dominated by an m = 1 spiral structure, as has been documented in the laboratory.

In the simulation, the power spectrum of the fluctuations exhibits a relative broadening, but its shape is masked by a numerical noise floor that follows a power-law frequency dependence of the form \( f^{-2} \). This is in contrast to the broadband frequency spectrum observed in the laboratory. The experiments display an exponential frequency dependence, i.e., the power spectrum is proportional to \( \exp(-4\pi f\tau) \), where the time scale \( \tau \) corresponds to the width of Lorentzian-shaped temporal pulses. It is surmised that the difference in the spectral behavior, aside from the numerical noise floor, arises from the inability of the simulation to arrive at an equilibrium in which the radial transport losses are balanced by the input from the heating source.

The exponential power spectrum observed in the LAPD heat transport experiments has been shown to be a signature of the underlying chaotic dynamics associated with ExB flows. Thus, the question arises about the nature of the dynamics in the simulation since the power spectrum lacks a clear exponential frequency dependence. To explore this issue, a C-H plane analysis is applied, as shown in Fig. 14. It is found that the signals from the simulation occupy the same chaotic region in the C-H plane as the measurements from the heat transport experiment, found in Fig. 6 of Ref. 23. The inference is that the underlying dynamics sampled by the simulation during the finite length of time corresponding to the saturation stage, \( \Omega_r t \approx 200 - 400 \), are chaotic.

A unique capability of the simulation is the visualization of the orbits followed by a select group of tagged particles. The insight gained from this tool is that the orbits of initially adjacent particles separate exponentially in time during the stage leading to the saturation of the drift-Alfvén waves. This behavior, often viewed as a definition of a chaotic process, provides independent support for the result of the C-H plane analysis in which the fluctuations are found to be chaotic.

The modest relaxation of the temperature profile seen in the simulation during the stage, where the drift-Alfvén waves have been identified to cause chaotic motion, is representative of the relaxation observed in the experiments. But the...
rapid collapse that arises in the simulation from the $k_z = 0$ convective cells has not been observed, as might be expected because of the open axial boundary conditions encountered in the laboratory.

What differs fundamentally between the simulation results and the laboratory studies is the nature of the asymptotic state. In the experiments, a transport equilibrium is reached in which chaotic advection by ExB flows is balanced against the power input from the heating source. And, of course, local thermal equilibrium is maintained due to Coulomb collisions. In the simulation, two widely different asymptotic states can be attained depending on the presence of $k_z = 0$ modes. Without them, as is closer to the experimental situation, the simulation reaches a state in which there is a broadened temperature profile, resembling that observed in the laboratory, but its essence is closer to a Bernstein-Green-Kruskal (BGK) equilibrium. In this state, finite amplitude drift-Alfvén waves simultaneously coexist with the broadened temperature profile, and with a modified parallel velocity distribution function. The asymptotic state reached in the presence of $k_z = 0$ modes is radically different. It is reached by a fission process that results in a collection of convective cells and smaller-scale filaments.

In regard to nonlinear processes, it has been found that drift-Alfvén waves can drive convective cells to a relative large level in the absence of zero-order density gradients. Surprisingly, the combination of these two mode structures has been found to result in the generation of radially expanding heat pulses, somewhat reminiscent of the blob structures frequently observed at the edge of fusion devices.

VII. CONCLUSIONS

A 3D electromagnetic, gyrokinetic particle code has been found to quantitatively reproduce some of the principal features observed in basic experiments on electron heat transport. This success has been achieved in spite of some extreme differences with the experimental environment such as neglect of collisions, periodic boundary conditions, and lack of an external heat source. It is thus suggestive that it would be valuable to undertake a future simulation study in which such effects are included.

The asymptotic state of rapid temperature profile relaxation, resulting from the combined effect of drift-Alfvén waves and convective cells, warrants a dedicated theoretical and simulation investigation. It would also be of interest to search for experimental arrangements that may capture some of these interesting processes.

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