Hard x-ray tomographic studies of the destruction of an energetic electron ring

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(Received 10 October 2012; accepted 25 April 2013; published online 15 May 2013)

A tomography system was designed and built at the Large Plasma Device to measure the spatial distribution of hard x-ray (100 KeV-3 MeV) emissivity. The x-rays were generated when a hot electron ring was significantly disrupted by a shear Alfvén wave. The plasma is pulsed at 1 Hz (1 shot/s). A lead shielded scintillator detector with an acceptance angle defined by a lead pinhole is mounted on a rotary gimbal and used to detect the x-rays. The system measures one chord per plasma shot using only one detector. A data plane usually consists of several hundred chords. A novel Dot by Dot Reconstruction (DDR) method is introduced to calculate the emissivity profile from the line integrated data. In the experiments, there are often physical obstructions, which make measurements at certain angles impossible. The DDR method works well even in this situation. The method was tested with simulated data, and was found to be more effective than previously published methods for the specific geometry of this experiment. The reconstructed x-ray emissivity from experimental data by this method is shown. © 2013 AIP Publishing LLC. [http://dx.doi.org/10.1063/1.4804354]

I. INTRODUCTION

A recent experiment performed at Large Plasma Device (LaPD) at the University of California, Los Angeles has demonstrated the significant disruption of a magnetic mirror trapped hot electron ring by a shear Alfvén wave. This has a potential application to radiation belt remediation techniques. In this experiment (Fig. 1(a)), a population of energetic electrons was generated by 2nd harmonic electron-cyclotron-resonance heating (ECRH), and trapped by a magnetic mirror field \(B_{\text{max}} = 800 \text{ G}, B_{\text{min}} = 437 \text{ G}\). The magnetic axis is in the z direction. These trapped hot electrons drifted in the azimuthal direction and formed a hot electron ring in the magnetic mirror. Hard x-rays were generated when the hot electrons struck the machine wall or other metal objects in the vacuum chamber. Detrapping of the electrons occurred via a shear Alfvén wave launched by a rotating magnetic field antenna. Figure 1(b) shows such a wave launched by the same antenna in a uniform magnetic field of 1 kG. The shear Alfvén wave propagated along the magnetic field, with two current channels (red and blue in Fig. 1(b)) rotating in the right-handed direction with respect to the background \(B\) field.

With the application of the shear Alfvén wave, a rapid increase in the hard x-ray signal was observed (Fig. 1(c)), indicating the release of energetic electrons from the mirror trap. The hard x-rays were originally detected by an un-collimated scintillator detector with no spatial resolution.

Because the x-rays are traces of the de-trapped energetic electrons, the x-ray emissivity profile can reveal the spatial distribution of the released hot electrons. The obvious difficulty of focusing hard x-rays precludes direct imaging. Collimated line-integrated emission data can be measured. The well-known Abel inversion technique for line-integrated data is restricted to axially symmetric geometries, which we do not expect in this case due to the asymmetric injection of ECRH. In order to invert line integrated measurements to obtain the local emissivity profile without any assumption of symmetry, some method of computed tomography (CT) is required.

At present, the most common application of CT is in medical imaging. The accuracy and efficiency of medical CT scanners has improved significantly since 1967 when it was first introduced. X-ray emissivity tomography has been successfully applied to plasma diagnostics, and many different reconstruction algorithms have been developed. In this paper, we report a tomography system developed to measure the x-ray emissivity profile in the LaPD. In Sec. II, the hardware of the tomography system is described. In Sec. III, a novel method to reconstruct the emissivity profile from the line integrated measurement is introduced. In Sec. IV, different methods are applied to simulated test functions and the experimental measurement. The methods are compared to the one developed here.

II. DESIGN OF HARDWARE

The hard x-rays produced in this experiment were energetic enough (>100 keV) to penetrate the 3/8 in. thick stainless steel wall of the vacuum chamber and to be measured outside the vacuum chamber. These measurements can be adjusted to get absolute intensities by considering the transparency of the chamber wall as a function of the x-ray energy. Therefore, we designed a tomographic system which works outside the vacuum chamber for this experiment.

The detection unit is an integrated scintillator detector, which contained a 7.62 cm diameter \(\times\) 7.62 cm long cylindrical NaI(Tl) crystal, a Photo-Multiplier Tube (PMT) directly optically coupled to the crystal, and a layer of mu-metal to magnetically shield the PMT. The detector was calibrated with \(^{137}\text{Cs}\) for pulse height analysis. The detector response is linear in x-ray energy in the range of interest from 100 keV to 3 MeV.

REVIEW OF SCIENTIFIC INSTRUMENTS 84, 053503 (2013)
FIG. 1. (a) A schematic diagram of the experiment. (b) Isosurface of current from a shear Alfvén wave generated by the RMF antenna. The white arrows are magnetic field vectors of the wave. The shear Alfvén wave propagates mostly along the magnetic field lines. The image on the bottom shows the underside of the wave. (c) The un-collimated x-rays measured with or without the presence of a 115 kHz shear Alfvén wave (SAW) launched from $t_{23}$ ms to $t_{23.9}$ ms. The ECRH is on from $t_{0}$ ms to $t_{30}$ ms.

To get a line of sight integrated measurement of the x-rays, the detector is encased in lead shielding with an opening for a collimator (Fig. 2). The thinnest part of the lead is 5.8 cm, which passes less than 6% of x-rays with energy in the range 100 keV-3 MeV. The lead was cast into an iron tube, which also provided additional magnetic shielding for the PMT. As a test, the detector was placed in a 1000 G ambient field at arbitrary directions, and no effect was found in the detector output; this field is substantially larger than that where the detector is located in the experiment. The collimator was made of lead with a 10.2 cm long cylindrical tunnel in the center. The inner part of the collimator was replaceable to change the acceptance angle in different experimental conditions. Three replacement collimators were made, with hole-diameters of 12.7 mm, 6.4 mm, and 3.2 mm.

The LaPD is a cylindrical device, with the rotational and magnetic axis in the z-direction (Fig. 1(a)). Tomography planes were transverse to the axis (x-y planes). Line integrations of x-ray emissivity are acquired with different $(t, \theta)$ values in each transverse plane to produce a tomogram, as shown in Fig. 4(a), where $t$ is the impact parameter of the chord offset from the vacuum chamber center, and $\theta$ is the angle from the x-axis. Since the experiment was highly reproducible and was repeated once every second, numerous line projections were measured with only one movable detector. There are 5 independently changeable parameters associated with any one projection $(x, y, z, \theta, \phi)$, where $x, y, z$ are the Cartesian coordinates of the detector position and $\theta, \phi$ are the polar and azimuthal angles of the projection direction (to be consistent with the previously described coordinate system, we now set the direction with $(\theta, \phi) = (0, 0)$ to be the x-direction). To achieve these parameters, a computer controlled rotation drive was designed and constructed (Fig. 3). It was fixed on a horizontal platform which can move vertically with 1 mm precision and was able to hold the 300 lb weight of the drive and the detector shielding mounted on it. For an axial tomogram (a slice transverse to the axis), $\phi$ was fixed at 0 and $z$ was fixed at one axial location. In this experiment, $x$ was also fixed and $y$ was changed by raising and lowering the platform (Fig. 3). $\theta$ was changed by the rotation drive with precision of 0.02°.

The projection parameter $(t, \theta)$ is limited by several factors. Clearance of all parts of the rotation drive from the LaPD magnets and its supporting structures requires $|x| \geq 90$ cm.

FIG. 2. Detector shielding scheme. The x-ray detector consists of a NaI(Tl) scintillator and PMT, and is shielded with at least 5.8 cm of lead, which passes only 1% of 1 MeV x-rays. A replaceable collimator was added to the shielding.

FIG. 3. A drawing of the rotation drive with the detector shielding mounted on it. The rotation drive was moved by a computer controlled data acquisition system. The whole assembly weighed about 300 lb, and was mounted on a vertically movable platform.
The range of the vertical motion of the platform sets \(-57\) cm \(\leq y \leq 69\) cm. The rotation drive has a motion range which limits \(-57.5^\circ < \theta < 56.3^\circ\). The achievable \((t, \theta)\) is plotted in Fig. 4(b). The x-ray emissivity is confined inside the vacuum chamber which has a radius of 50 cm, therefore only the region with \(|t| \leq 50\) cm is considered.

FIG. 4. (a) A drawing of the experiment geometry in the transverse x-y plane. \(t\) is the perpendicular distance from the origin to the chord. The sign of \(t\) is defined as the sign of the y-intercept. (b) A map of \((t, \theta)\) coverage by the tomography system. The hatched area can be covered with the detector moving only on the positive x side of the vacuum chamber. The uniformly shaded area is added if the detector is moved to the negative x side of the vacuum chamber.

III. RECONSTRUCTION ALGORITHMS

Many reconstruction methods are available to produce a tomogram from a set of projection measurements. In an ideal case where an infinite set of chords is measured precisely, the outcome of these different methods should converge to the same result. However, given a finite set of projections the tomogram is not uniquely determined, and different methods give different results.

Over the past few decades numerous reconstruction methods have been introduced and repeatedly performed, largely driven by medical applications, and new methods are likely to emerge with the growth of computational power. We now briefly list a few of the methods as examples, detailed description of these methods can be found in the references: (a) analytic methods based on Fourier transform;\(^5\),\(^12\) (b) Back projection;\(^5\),\(^12\) (c) Least-squares fitting;\(^6\)–\(^10\) (d) ART (Algebraic Reconstruction Technique);\(^11\),\(^12\) (e) Regularization operator method.\(^10\) Here, we introduce a Dot by Dot Reconstruction (DDR) method that we developed, and have found to be an improvement over the existing methods for this study.

The rest of this section describes the DDR method and is organized into 4 parts. The first part discusses a concept essential to the DDR method: presenting the integrated measurements in the projection parameter space. In the 2nd part, the tomography reconstruction problem is re-introduced from a new perspective that directly leads to the DDR method. In the 3rd part, a point-selection criteria is defined, which is calculated using the integrated measurement, and is used throughout the DDR reconstruction process. Next, the full DDR reconstruction procedure is described (a simple example is included in the Appendix). Some discussions of the procedure are included in the 4th part of this section.

A. Understanding \(S(t, \theta)\): Measurements presented in projection parameter space

The x-ray emissivity distribution function \(f(\vec{r})\) is the x-ray power emitted per unit volume at spatial location \(\vec{r}\), with units of \(\text{W/m}^3\). An actual detector measures emission from a finite span of angles. Therefore, the measurement

\[
S(t, \theta) = \frac{1}{4\pi} \int f(\vec{r})\Omega(\vec{r}, t, \theta) d\vec{r},
\]

where \(\Omega(\vec{r}, t, \theta)\) is the solid angle that the detector subtends at the spatial point \(\vec{r}\), when the detector is collimated to the \((t, \theta)\) direction (as in Fig. 4(a)). If the collimator is narrow enough and the detector is far enough from the volume of interest, the detector can be considered as an ideal collimated detector, for which \(\Omega\) is uniform along the line of sight \(L_{(t, \theta)}\) in the volume of interest. \(L_{(t, \theta)}\), as shown in Fig. 4(a), is described by parametric equations

\[
\begin{align*}
    x &= l \cos \theta - t \sin \theta, \\
    y &= l \sin \theta + t \cos \theta,
\end{align*}
\]

Here, \(l\) is the length along the chord. If we assume the emissivity is uniform in the z-direction, we can write a 2D emissivity as \(f(x, y)\), and the measurement \(S(t, \theta)\) in units of \(\text{W/m}^2\) can be defined as the line integral of \(f(x, y)\) along \(L_{(t, \theta)}\)

\[
S(t, \theta) = \int_{L_{(t, \theta)}} f(x, y) dl
= \int f(l \cos \theta - t \sin \theta, l \sin \theta + t \cos \theta) dl.
\]

For a single point source at \((x_0, y_0)\) with intensity \(\varepsilon\), the emissivity is

\[
f(x, y) = \varepsilon \delta(x - x_0) \delta(y - y_0).
\]

From (2) and (3), and using \(\delta(ax) = \delta(x)/|a|\) for non-zero \(a\), the measurement \(S_0(t, \theta)\) arising from this one-point emission
profile is
\[ S_0(t, \theta) = \int \varepsilon \delta(l \cos \theta - t \sin \theta - x_0) \]
\[ \times \delta(l \sin \theta + t \cos \theta - y_0)dl \]
\[ = \int \varepsilon \frac{\delta(l - \frac{t \sin \theta + x_0}{\cos \theta})}{\cos \theta} \delta(l \sin \theta + t \cos \theta - y_0)dl \]
\[ = \frac{\varepsilon}{\cos \theta} \delta \left( \frac{t \sin \theta + x_0}{\cos \theta}, \sin \theta + t \cos \theta - y_0 \right) \]
\[ = \varepsilon \delta(y_0 \cos \theta - x_0 \sin \theta - t), \]
where \( \theta \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) \). In the case of \( \theta = \frac{\pi}{2} \), (4) still holds since
\[ S_0 \left( t, \frac{\pi}{2} \right) = \int \varepsilon \delta(-t - x_0) \delta(l - y_0)dl \]
\[ = \varepsilon \delta(-t - x_0) = \varepsilon \delta(y_0 \cos \theta - x_0 \sin \theta - t). \]

Therefore, the measurement of a point source at \((x_0, y_0)\) is represented by a curve \( \Gamma_{(x_0, y_0)} \) in the \((t, \theta)\) projection parameter space. The curve, labeled by the point \((x_0, y_0)\), can be expressed by writing \( t \) as a function of \( \theta \)
\[ t_{(x_0, y_0)}(\theta) = y_0 \cos \theta - x_0 \sin \theta, \quad \text{where} \quad \theta \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right). \]

Only when the detector’s line of sight \((t, \theta)\) satisfies (5) can it detect emission from the point source at \((x_0, y_0)\). For example, for a point source shown in Fig. 5(a) located at \((x_0, y_0) = (1, 1)\), the detector will see the source on any line of sight that has a projection parameter that lies on the dark curve in Fig. 5(b), such as \((t, \theta) = (1, 0)\). Such \((t, \theta)\) combinations, which represent all chords going through the point source, form the curve \( \Gamma_{(x_0, y_0)} \) which is described by (5).

Because the above transformation is linear in \( f \), a collection of point sources at locations of \( P = \{(x_i, y_i)\} = 1, \ldots, N \) with uniform intensity \( \varepsilon \) in image space \((x, y)\) have a chord integrated measurement \( S_P(t, \theta) \) which is the superposition of the curves \( \Gamma_{(x_i, y_i)} \)
\[ S_P(t, \theta) = \sum_i \varepsilon \delta(y_i \cos \theta - x_i \sin \theta - t). \]

B. The problem of tomography reconstruction from a new perspective

The tomography reconstruction problem in its most common form is simply to calculate an emission profile \( f(x, y) \) corresponding to a given measurement \( S(t, \theta) \). It is the inverse process of the previously discussed process of getting measurements by integrating an emission profile.

The emissivity \( f(x, y) \) is a continuous function, but we assume it can be approximated by a set of emitting points with a small and uniform intensity \( \varepsilon \). These emitting points can overlap to account for different intensities at different locations. As above, we designate the locations of these emitting points by \( P \). In this perspective, the tomography reconstruction problem can be stated as: given the chord integrated measurement \( S_P(t, \theta) \) of a set of point sources with uniform intensity \( \varepsilon \), find the locations of all the points in this set \( P \).

As previously discussed, the measurement of many point sources presented in \((t, \theta)\) space is the superposition of curves in the form of (6). In a reconstruction problem, we only have the left-hand side of (6) and wish to determine the right-hand side. Since each of these curves is labeled with the location of the point source, if we identify all the curves, then the locations of the point sources are found and the tomography problem is solved. The question now is how to identify these curves. In the following part, a point-selection criteria is defined, which compares the measurement with a single curve in the form of (5) generated by an arbitrary point \((x, y)\). With this criteria, the superimposed curves can be identified one at a time to reconstruct the emissivity profile.

C. The point-selection criteria and the procedure of DDR

For a chord integrated measurement \( S_P(t, \theta) \), the point-selection criteria \( I_{S_p} \) for an arbitrary point of interest \((x, y)\) is defined as the average value of \( S_P(t, \theta) \) over all possible angles of chords passing through the point \((x, y)\)
\[ I_{S_p}(x, y) = \frac{1}{\pi} \int_{-\pi/2}^{\pi/2} S_P(t_{(x, y)}(\theta), \theta)d\theta, \]
where \( t_{(x, y)}(\theta) \) is defined in (5). This criteria has a more intuitive meaning, which can be found by substituting (5) and (6) into (7)
\[ I_{S_p}(x, y) = \frac{1}{\pi} \int_{-\pi/2}^{\pi/2} \sum_i \varepsilon \delta(y_i \cos \theta - x_i \sin \theta - y \cos \theta + x \sin \theta)d\theta \]
\[ = \frac{1}{\pi} \sum_i \frac{1}{\sqrt{(x - x_i)^2 + (y - y_i)^2}}. \]

Note that every point in \( P \) represents the location of a point source of x-ray emission \( \varepsilon \). If one equates this to a point charge, then the integral \( I_{S_p}(x, y) \) is the electric potential from these charges at the point \((x, y)\), with a constant factor of \( \frac{1}{\pi} \). Therefore, one apparent way to reconstruct the emissivity \( f \) is to calculate \( I_{S_p}(x, y) \) for all \((x, y)\), which is equivalent to the electric potential field everywhere; then the reconstructed...
emissivity profile \( f_{rec}(x, y) \), which is equivalent to charge distribution, can be calculated by taking the Laplacian of the equivalent potential field

\[
f_{rec}(x, y) = -\frac{1}{4} \nabla^2 I_{Sp}(x, y).
\]  

(8)

This method requires measurement \( S \) on the full \((t, \theta)\) projection space in order to exactly calculate \( I_{Sp}(x, y) \). In the experiment, due to limitations described in Sec. II, the projection space can only be partially sampled (see Fig. 4(b)). So we define the generalized point-selection criteria \( I_{Sp}(x, y) \) as the integral only in the area where \( S \) is measured, e.g., \( \theta_1 < \theta < \theta_2 \):

\[
\tilde{I}_{Sp}(x, y) = \frac{1}{\theta_2 - \theta_1} \int_{\theta_1}^{\theta_2} S_P(t(x, y)(\theta), \theta) d\theta.
\]

In this case, there is not enough information for a unique reconstruction, and the following assertion is made: For two points \((x_A, y_A)\) and \((x_B, y_B)\), if \( \tilde{I}_{Sp}(x_A, y_A) > \tilde{I}_{Sp}(x_B, y_B) \), then it is more likely that the point \((x_A, y_A)\) belongs to \( P \) than the point \((x_B, y_B)\) does.

Consider a set of locations \( Q = \{(x_i, y_i)\} \), where it is possible to have a non-zero emissivity. Essentially this is an infinite set with all points in the image space, but as an approximation, we can choose a finite number of locations uniformly distributed in the area of interest, so that any point in \( P \) can be approximated by a point in \( Q \) within a given finite spatial resolution (because of this, the points in \( Q \) can be considered as candidate points). The DDR procedure determines which of the points in \( Q \) are the best suited to reconstruct \( P \). Because the points in \( P \) can overlap, we allow any candidate point in \( Q \) to be chosen multiple times. As a reminder, \( P \) is the set of locations of the unit intensity point sources, whose elements can appear more than once (strictly speaking, \( P \) is a multiset), and is the unknown set to be reconstructed. \( Q \) is the set of all possible emitting locations, in which no two elements are same.

To begin with, assume \( P \) is not empty, there is at least one candidate point in \( Q \) that is also in \( P \). To find this point, \( \tilde{I}_S \) is calculated for every point in \( Q \), and the point \( q_1 \) that has the maximum \( \tilde{I}_S \) is chosen. Only one candidate point can be chosen, because (a) the next best choice may represent the same point in \( P \), and (b) the next point to be chosen may be a repeat of the same candidate point. To exclude these possibilities, we redefine the set to be reconstructed as \( P^{(1)} = P - \{q_1\} \), thus the measurement data need to be updated to exclude the portion of the measurement resulting from \( q_1 \), i.e., \( S^{(1)} = S - S_{q_1} \). Assuming \( S^{(1)} > 0 \) (meaning \( P^{(1)} \) is not empty), using this updated measurement we calculate \( \tilde{I}_S^{(1)} \) for every point in \( P \), then a second point \( q_2 \) (which can be same as \( q_1 \) if necessary) can be chosen from \( Q \) by searching for the maximum \( \tilde{I}_S^{(1)} \), and the measurement can be updated again: \( S^{(2)} = S^{(1)} - S_{q_2} \). This process can be repeated to find a set of points \( \{q_1, q_2, \ldots \} \) which represent the reconstructed emissivity. The stopping condition at the ith iteration is the newly updated measurement \( S^{(i)} = 0 \), which means \( P^{(i)} \) is empty and all points in \( P \) are found. In practice, this condition can be implemented as a test that the average value of \( S^{(i)} \) is sufficiently small. To better illustrate the procedure, a simple example is included in the Appendix.

D. Some discussion about DDR

In summary, we try to decompose the measured \( S(t, \theta) \) into curves with the form of (6). In the reconstruction process, the curve which represents the maximum contribution to \( S \) is treated as the most “likely” one at a given stage in the reconstruction, and the corresponding point in image space is chosen to be included in the reconstruction. When the reconstruction process is complete, we have found a set of curves in \((t, \theta)\) space corresponding to a known set of emitting points in the image space. Since the sum of these curves approximates \( S \) over the area where \( S \) is measured, the set of corresponding points is a solution to the tomography problem.

For an actual measurement with non-ideal \( \Omega(\vec{r}, t, \theta) \) in (1), the measurement \( S_0(t, \theta) \) of a point source of intensity \( \epsilon \) at \((x_0, y_0)\) presented in the \((t, \theta)\) space is not a line of uniform intensity as in (4), but a curve with finite width and varying intensity. This actual \( S_0(t, \theta) \) can be calculated using (1) once \( \Omega(\vec{r}, t, \theta) \) is known, which can be calculated from the collimator geometry and the detector location. In this case, the calculation of \( I_{Sp}(x, y) \) can be generalized to

\[
I_{Sp}(x, y) = \frac{1}{\pi \epsilon^2} \int_{-\pi/2}^{\pi/2} \int_{-\infty}^{\infty} S_P(t, \theta) S_0(t, \theta) dt d\theta.
\]  

(9)

In the limit of an idealized case, by substituting \( S_0(t, \theta) \) in the form of (4) and (9) gives the same result as the original definition in (7).

As mentioned above, this method makes an explicit approximation, that the continuous 2D function to be reconstructed is approximated by a set of points. In other words, the emissivity function is approximated by a set of emitting dots, and the small intensity of each of these dots \( \epsilon \) is the “unit emissivity.” The amplitude of \( f \) at a position is approximated by the number of points in the vicinity of this position. \( \epsilon \), which sets the amplitude resolution, should be sufficiently small for the desired smoothness of the reconstructed emission profile, but reducing its value requires increased computation time. Figure 6 shows a demonstration of this method with an arbitrary test function, which is reconstructed with \( \sim 10^4 \) points. The maximum value of the test function max \( f \) = 1, and the amplitude of each dot \( \epsilon = 0.05 \). \( S(t, \theta) \) of the reconstructed image (Fig. 6(e) bottom) agrees with the original measurement (Fig. 6(a) bottom) within 0.5% error.

This method has several features that make it attractive for reconstructions in this experiment. First, the method works with limited measurement. Second, non-negativity of the emissivity is ensured by the reconstruction procedure, since only positive numbers are added to it. Third, to calculate the line integral the integration area should be well sampled. This requirement can be satisfied in this experiment, as in Fig. 4(b) the shaded area can be sampled continuously. Finally, this method empirically produces less undesired artifacts compared to some other methods with the geometry used in this experiment. Comparisons of reconstruction results by different methods will be shown in Sec. IV.

The computation time is generally longer for DDR compared to most of the other methods tested by the authors. If \( S \) is sampled on \( N_e \) points (each point is a line of sight
FIG. 6. (a) An arbitrary test 2D emissivity. Using DDR method, $2.8 \times 10^4$ points are found one by one, to reconstruct the emissivity. The course of the reconstruction is shown in (b)–(e), which contains the first $0.2 \times 10^4, 1 \times 10^4, 2 \times 10^4$, and $2.8 \times 10^4$ points. The images on the top row are emissivity profiles, and their corresponding line of sight integrated measurements are shown on the bottom.

IV. RESULTS FROM SIMULATED DATA AND EXPERIMENT DATA

The experiment has a limited set of measurements in the $(t, \theta)$ projection space. In a practical sense, it is helpful to test the reconstruction algorithms with the projection geometry used in the experiment. A Gaussian $f(x, y) = \exp(-\frac{(x+20)^2 + y^2}{200})$ was chosen as the test function (plotted in Fig. 7(a)), and was numerically “measured” with the same geometry as used in the experiment (see Fig. 7(b)).

Five different published procedures$^{5-12}$ were tested with the simulated measurement for comparison with the proposed technique. In each case, the reconstructed emissivity function $f_{rec}$ has undesired artifacts, as shown in Fig. 8. For example, some $f_{rec}$ have negative values. Another significant artifact is displacement of the peak position. Also in some $f_{rec}$ artificial peaks are generated separate from the actual peak. These artifacts arise from the limitations of the measurement technique, and within the context of any one reconstruction it is impossible to separate them from the “real” emissivity. The DDR method has the least artifacts with this measurement geometry.

The DDR method has also been applied to the experimental measurements. The LaPD plasma is generated from a pulsed dc (direct current) discharge between a cathode and a molybdenum mesh anode at one end of the machine$^1$ as shown in Fig. 1(a). The anode is about 10.75 m from the trapped electron ring. X-rays were generated when energetic electrons strike the anode, and a tomogram was measured on the anode plane. In this case, 549 projections were measured (the distribution is shown in Fig. 7(b)), each averaged over 60 shots. The emissivity is reconstructed using the DDR method, with the result shown in Fig. 9. The other techniques described above generated artifacts similar to those in Fig. 8, and were considered much less likely to be accurate representations of the x-ray emission, particularly those that require (nonphysical) negative emission from the reconstructed source.

In this experiment, a population of hot electrons was trapped by a magnetic mirror field, and formed a hot electron ring.$^2$ Due to the imperfect confinement of the magnetic
mirror, some hot electrons are slowly lost and x-rays are continuously generated on the anode, as shown in Fig. 9(a). The shear Alfvén wave dramatically scattered the fast electrons out of the mirror, and as a result the x-ray emissivity increased significantly in Fig. 9(b).

The line cut of the emissivity profile before the Alfvén wave appears to be double peaked (located at $x = \pm 7 \text{ cm}$). The fast electrons travel mainly along the magnetic field lines, and the magnetic field line at $x = \pm 7 \text{ cm}$ on the anode plane extended to $x = \pm 9.2 \text{ cm}$ in the center of the magnetic mirror. The peak on Fig. 9(b) is located at $x = -6 \text{ cm}$ which corresponds to $x = -7.9 \text{ cm}$ in the magnetic mirror and $x = -5.9 \text{ cm}$ at the RMF antenna.

ACKNOWLEDGMENTS

The authors are very grateful to S. Vincena for his help in integrating the tomography module that drove the tomography detector and acquired data to the LaPD main data acquisition system, to Z. Lucky and M. Drandell for their expert technical support, and to Professor K. Papadopoulos for valuable discussion. The authors also thank Dr. Y. Zhao for interesting discussions on medical CT imaging. This work is supported by the Office of Naval Research (ONR) and performed at the Basic Plasma Science Facility under ONR MURI award. The BaPSF is funded by the (U.S.) Department of Energy (DOE) and the National Science Foundation (NSF).

APPENDIX: A STEP BY STEP DEMONSTRATIVE EXAMPLE OF THE DDR PROCEDURE

The reconstruction procedure can be illustrated by a simple example. Suppose we have chord-integrated measurements (shown in Fig. 10(b)) of some unknown emissivity, and we will try to reconstruct the emissivity profile from this measurement. This unknown emissivity profile is shown in Fig. 10(a) for this demonstration, but during the reconstruction process we do not use any information from it.

We start with the assumption that the emissivity profile can be approximated by a collection of emitting points with uniform intensity. (Readers who still need to be convinced can read Sec. III B, or take a look at the works of the French pointillist painter Paul Signac.) This unknown collection of points is designated by $P$ in this example. The unit emission intensity $\varepsilon$ of the point sources needs to be chosen artificially. $\varepsilon$ should be small enough to be able to reflect the smallest difference in the emissivity, in this case we can choose $\varepsilon = 1$. Note the emissivity and the integrated measurement have different units that are different by a constant factor.

The authors are very grateful to S. Vincena for his help in integrating the tomography module that drove the tomography detector and acquired data to the LaPD main data acquisition system, to Z. Lucky and M. Drandell for their expert technical support, and to Professor K. Papadopoulos for valuable discussion. The authors also thank Dr. Y. Zhao for interesting discussions on medical CT imaging. This work is supported by the Office of Naval Research (ONR) and performed at the Basic Plasma Science Facility under ONR MURI award. The BaPSF is funded by the (U.S.) Department of Energy (DOE) and the National Science Foundation (NSF).
FIG. 10. (a) A model emissivity profile. It can be approximated by 3 point sources with intensity of 1, located at \( \{ (-1,1), (1,1), (1,1) \} \). The collection of these point sources locations is designated by \( P \). (b) Chord-integrated measurement of an unknown emissivity profile, which is in fact (a). (c) Locations of candidate points. Candidate points are artificially chosen possible locations of emission.

The next step is to choose a set of possible emission locations for the reconstruction. This set, known as the candidate set and designated by \( Q \), is discussed in more detail in Sec. III C. From the measurement in Fig. 10(b), all measurements are zero if \( |t| \) (the distance from the chord to the origin, see Fig. 4(a)) is larger than 1.5, therefore all the emitting
sources are within \( r = 1.5 \) from the origin. We make a blind guess that the emission can only come from 9 points shown in Fig. 10(c), i.e., \( Q = \{(-1, 1), (0, 1), (1, 1), (-1, 0), (0, 0), (1, 0), (-1, -1), (0, -1), (1, -1)\} \). Generally, the more points in \( Q \), the higher the chance that a real source of emission is near one of these points, hence the better the reconstruction approximates the original emissivity. Here, we try to keep the demonstration simple. An example with a different guess for \( Q \) is given at the end of this appendix, to show the perturbation to the reconstruction result.

Next, the DDR procedure will choose points from \( Q \) to be included in the reconstruction. At the end of the procedure, a reconstructed emissivity profile will be created which should be the same as (or similar enough to) the original emissivity in Fig. 10(a). The procedure is iterative and is described step by step as following:

1. **1st step in 1st iteration**: Choose a point in \( Q \) that most likely belongs to \( P \). This is done by choosing the point with the largest point-selection criteria, calculated by Eq. (7) (details of this criteria are discussed in Secs. III C and III D). Here, \( S \) is the chord-integrated measurement we have, shown in shades of blue in Figs. 10(b) and 11(a); \((x, y)\) are coordinates of points in \( Q \), which define the integration path \( t_{1, x, y}(\theta) = y \cos \theta - x \sin \theta \) shown in Fig. 11(a) as dotted lines for all 9 points in \( Q \). \( S \) integrated along the path generated by \((x, y) = (1, 1)\) gives the largest value, so we choose \((1, 1)\) as the first point to be included in the reconstruction.

2. **2nd step in 1st iteration**: Now that we have found the first point \((1, 1)\), we add a point source at \((1, 1)\) with intensity of \( \epsilon \) to the reconstruction (Fig. 11(b)).

3. **3rd step in 1st iteration**: Since we already found this point source, the measurement \( S \) needs to be updated to exclude the contribution from this point source, so that in the following reconstruction procedure this point source would not be considered again. This is done by calculating \( S^{(1)}(t, \theta) = S(t, \theta) - S_{(1,1)}(t, \theta) \), where \( S(t, \theta) \) is the measurement at the beginning of this iteration (shown in Fig. 11(a)), and \( S_{(1,1)}(t, \theta) \) is the measurement of this point source (shown in Fig. 11(c)). The updated measurement \( S^{(1)}(t, \theta) \) is shown in Fig. 11(d).

4. **4th step in 1st iteration**: Check if the updated measurement \( S^{(1)}(t, \theta) \) is zero. In this case, the total value in Fig. 11(d) is greater than zero, meaning there are still point sources to be found, the procedure should not stop now.

So far, one point source has been found by using \( S(t, \theta) \). Next, another point can be found by substituting \( S(t, \theta) \) with \( S^{(1)}(t, \theta) \) and repeat the procedure:

1. **1st step in 2nd iteration**: Calculate \( I_S(x, y) = \frac{1}{\pi} \int_{-\pi/2}^{\pi/2} S^{(1)}(t_{x, y}, \theta) \, d\theta \) for all \((x, y)\) in \( Q \). \( S^{(1)}(t, \theta) \) and the integration path is shown in Fig. 11(e). Along the path generated by \((1,1)\) and \((-1,1)\) gives the maximum value of \( I_S(x, y) \) among the 9 candidate points in \( Q \). Either one can be chosen at this point. We make an arbitrary choice of \((-1,1)\).

2. **2nd step in 2nd iteration**: Add a point source at \((-1,1)\) with intensity of \( \epsilon \) to the reconstruction (Fig. 11(f)).

3. **3rd step in 2nd iteration**: Update measurement \( S^{(2)}(t, \theta) = S^{(1)}(t, \theta) - S_{(-1,1)}(t, \theta) \), where \( S^{(1)}(t, \theta) \) is the measurement at the beginning of this iteration (shown in Fig. 11(e)), and \( S_{(-1,1)}(t, \theta) \) is the measurement of this point source (shown in Fig. 11(g)). The updated measurement is shown in Fig. 11(h).

4. **4th step in 2nd iteration**: Check if the updated measurement \( S^{(2)}(t, \theta) \) is zero. The total value in Fig. 11(h) is greater than zero, the procedure continues and the next iteration is started.

1. **1st step in 3rd iteration**: Calculate \( I_S(x, y) = \frac{1}{\pi} \int_{-\pi/2}^{\pi/2} S^{(2)}(t_{x, y}, \theta) \, d\theta \) for all \((x, y)\) in \( Q \), and choose \((1,1)\) from the 9 points in \( Q \) (Fig. 11(i)).

2. **2nd step in 3rd iteration**: Add a point source at \((1,1)\) with intensity of \( \epsilon \) to the reconstruction (Fig. 11(j)).

3. **3rd step in 3rd iteration**: Update measurement \( S^{(3)}(t, \theta) = S^{(2)}(t, \theta) - S_{(1,1)}(t, \theta) \). \( S^{(2)}(t, \theta) \) is shown in Fig. 11(i), \( S_{(1,1)}(t, \theta) \) is shown in Fig. 11(j), \( S^{(3)}(t, \theta) \) is shown in Fig. 11(l).

4. **4th step in 3rd iteration**: Check if the updated measurement \( S^{(3)}(t, \theta) \) is zero. The total value in Fig. 11(l) is zero, meaning there is no unknown point source left. The procedure stops here, and the emissivity is fully reconstructed by the latest reconstruction (Fig. 11(j)).
The blind guess of the candidate points \( Q \) we made is not a bad one, since it contains the two exact emitting locations. In a not so lucky case, consider another blind guess of \( Q \) shown in Fig. 12(a), in which all points are shifted 0.25 units in x-direction. With this guess, the process described above can still be performed in the same way, and the result is shown in Fig. 12(b). Compare to the target profile (Fig. 10(a)), the reconstruction is offset by 0.25. This result would be acceptable if the desired spatial resolution is larger than 0.25. With this candidate set, the points are uniformly distributed with grid spacing \( a = 1 \). In the worst case where the emitting point is between all candidate points, the reconstruction can be off by 0.7 as shown in Fig. 12(c). In order to improve the accuracy of the reconstruction, a general way is to have more points included in the candidate point set \( Q \). As indicated in Fig. 12(c), with a spatial resolution requirement of \( \delta \), the distance between the uniformly distributed candidate points \( a \) should be less than \( \sqrt{2\delta} \).

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